

Lesson 16

WHY NORMAL SUBGROUPS ARE SO IMPORTANT – ANSWERS

Let $Q = \{ (), (1,2,5,6)(3,8,7,4), (1,3,5,7)(2,4,6,8), (1,4,5,8)(2,7,6,3), (1,5)(2,6)(3,7)(4,8), (1,6,5,2)(3,4,7,8), (1,7,5,3)(2,8,6,4), (1,8,5,4)(2,3,6,7) \}$ be the quaternion group.

1. If $H = \left\{ \begin{array}{l} () \\ (1,3,5,7)(2,4,6,8) \\ (1,5)(2,6)(3,7)(4,8) \\ (1,7,5,3)(2,8,6,4) \end{array} \right\}$, then find the two right cosets of H in Q .

$$H = \left\{ \begin{array}{l} () \\ (1,3,5,7)(2,4,6,8) \\ (1,5)(2,6)(3,7)(4,8) \\ (1,7,5,3)(2,8,6,4) \end{array} \right\}$$

$$H(1,2,5,6)(3,8,7,4) = \left\{ \begin{array}{l} ()(1,2,5,6)(3,8,7,4) \\ (1,3,5,7)(2,4,6,8)(1,2,5,6)(3,8,7,4) \\ (1,5)(2,6)(3,7)(4,8)(1,2,5,6)(3,8,7,4) \\ (1,7,5,3)(2,8,6,4)(1,2,5,6)(3,8,7,4) \end{array} \right\} = \left\{ \begin{array}{l} (1,2,5,6)(3,8,7,4) \\ (1,8,5,4)(2,4,3,6) \\ (1,6,5,2)(3,4,7,8) \\ (1,4,5,8)(2,7,6,3) \end{array} \right\}$$

2. If $H = \left\{ \begin{array}{l} () \\ (1,2,5,6)(3,8,7,4) \\ (1,5)(2,6)(3,7)(4,8) \\ (1,6,5,2)(3,4,7,8) \end{array} \right\}$, then find the two right cosets of H in Q .

$$H = \left\{ \begin{array}{l} () \\ (1,2,5,6)(3,8,7,4) \\ (1,5)(2,6)(3,7)(4,8) \\ (1,6,5,2)(3,4,7,8) \end{array} \right\}$$

$$H(1,3,5,7)(2,4,6,8) = \left\{ \begin{array}{l} ()(1,3,5,7)(2,4,6,8) \\ (1,2,5,6)(3,8,7,4)(1,3,5,7)(2,4,6,8) \\ (1,5)(2,6)(3,7)(4,8)(1,3,5,7)(2,4,6,8) \\ (1,6,5,2)(3,4,7,8)(1,3,5,7)(2,4,6,8) \end{array} \right\} = \left\{ \begin{array}{l} (1,3,5,7)(2,4,6,8) \\ (1,4,5,8)(2,7,6,3) \\ (1,7,5,3)(2,8,6,4) \\ (1,8,5,4)(2,3,6,7) \end{array} \right\}$$

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3. If $H = \left\{ \begin{array}{l} () \\ (1,4,5,8)(2,7,6,3) \\ (1,5)(2,6)(3,7)(4,8) \\ (1,8,5,4)(2,3,6,7) \end{array} \right\}$, then find the two right cosets of H in Q .

$$H = \left\{ \begin{array}{l} () \\ (1,4,5,8)(2,7,6,3) \\ (1,5)(2,6)(3,7)(4,8) \\ (1,8,5,4)(2,3,6,7) \end{array} \right\}$$

$$H(1,3,5,7)(2,4,6,8) = \left\{ \begin{array}{l} ()(1,3,5,7)(2,4,6,8) \\ (1,4,5,8)(2,7,6,3)(1,3,5,7)(2,4,6,8) \\ (1,5)(2,6)(3,7)(4,8)(1,3,5,7)(2,4,6,8) \\ (1,8,5,4)(2,3,6,7)(1,3,5,7)(2,4,6,8) \end{array} \right\} = \left\{ \begin{array}{l} (1,3,5,7)(2,4,6,8) \\ (1,6,5,2)(3,4,7,8) \\ (1,7,5,3)(2,8,6,4) \\ (1,2,5,6)(3,8,7,4) \end{array} \right\}$$

4. Construct a multiplication table for the quotient group found in the previous problem.

Since

$$\begin{aligned} H(1,3,5,7)(2,4,6,8) \cdot H(1,3,5,7)(2,4,6,8) &= H(1,3,5,7)(2,4,6,8)(1,3,5,7)(2,4,6,8) \\ &= H(1,5)(2,6)(3,7)(4,8) = H \end{aligned}$$

our multiplication table is:

	H	$H(1,3,5,7)(2,4,6,8)$
H	H	$H(1,3,5,7)(2,4,6,8)$
$H(1,3,5,7)(2,4,6,8)$	$H(1,3,5,7)(2,4,6,8)$	H

Also, notice that this group is isomorphic to C_2 , the cyclic group of order 2.