Lesson 1

WHAT IS A GROUP – ANSWERS

1. Multiply the following permutations together. Multiply left to right.

   a. $(1,2,3,4)(3,4,5) = (1,2,4)(3,5)$

   b. $(1,2,3,4)(4,3,2,1) = (1)(2)(3)(4) = ()$

   c. $(1,2)(1,3)(1,4) = (1,2,3,4)$

   d. $(1,2)(1,2) = (1)(2) = ()$

2. Verify the associative law by performing the multiplication in two ways as indicated by the grouping symbols. Multiply left to right.

   $\left[(1,2,3,4)(2,4)\right](1,3,5) = (1,4)(2,3)(1,3,5) = (1,4,3,2,5)$

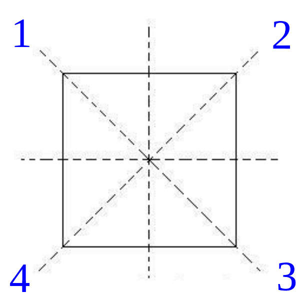
   $(1,2,3,4)\left[(2,4)(1,3,5)\right] = (1,2,3,4)(1,3,5)(2,4) = (1,4,3,2,5)$

3. If $a = (1,2,3)$, find a^2 and a^3.

   $a^2 = (1,2,3)(1,2,3) = (1,3,2)$

   $a^3 = a \cdot a \ = (1,2,3)(1,3,2) = (1)(2)(3) = ()$

4. If you rotate the square below through multiples of $90°$ or flip the square about one of the indicated axes of symmetry, then eight configurations are possible. Represent each configuration in terms of the corresponding cycle(s).



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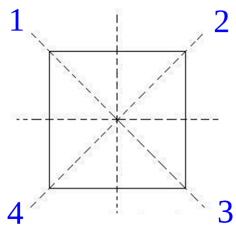
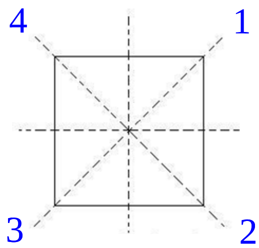
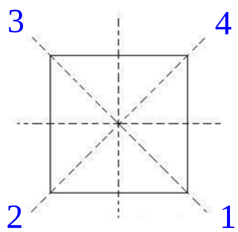


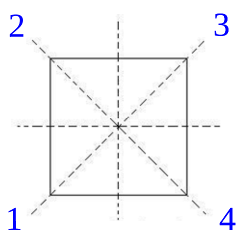
, $(1)(2)(3)(4) = ()$



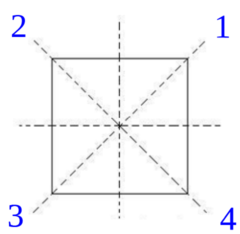
, $(1,2,3,4)$



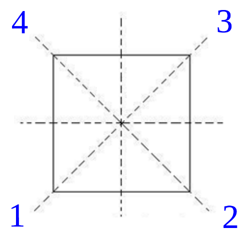
, $(1,3,4,2)$



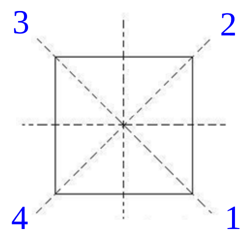
, $(1,4,3,2)$



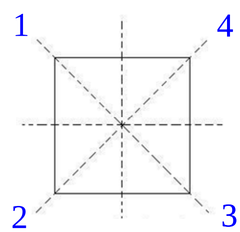
, $(1,2)(3,4)$



, $(1,4)(2,3)$



, $(1,3)$



, $(2,4)$

5. Explain why each of the following is not a group.

   a. The set of real numbers under subtraction.

      The associative law does not hold. For example, $(10-3)-2=7-2=5$, but $10-(3-2)=10-1=9$.

   b. The set of real numbers under multiplication.

      The identity element is 1, but 0 has no multiplicative inverse. In other words, there is no number 0^{-1} such that $0 \cdot 0^{-1} = 1$.

   c. The set of irrational numbers under multiplication.

      This set is not closed under multiplication. For example, $\sqrt{2} \cdot \sqrt{2} = 2$, and 2 is not an irrational number.

6. Identify two cycles from your physical world that are based upon symmetries or repetition of patterns, and identify one cycle that is based upon events that you repeat over time. Also, give the length of each cycle. The goal here is to realize that cycles are everywhere in your life!

   a. In the picture below there are four identical chairs. If I designate their positions as 1, 2, 3, and 4, then I can move the chair in position 1 to position 2, 2 to 3, 3 to 4, and 4 to 1. This gives us the cycle $(1,2,3,4)$ which has length 4. Also, the end result of moving the chairs is that nothing will look any different.

   b. In the picture below there are three identical wine glasses. If I designate their positions as 1, 2, and 3, then I can move the glass in position 1 to position 2, 2 to 3, and 3 to 1. This gives us the cycle $(1,2,3)$ which has length 3. Also, the end result of moving the glasses is that nothing will look any different.

   c. Every day I wake up in the morning and go to sleep at night. This defines a cycle in time of length 2, $(\text{wake up, go to sleep})$. I hope to repeat this cycle forever!