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DUH?

A differential equation involving a function of a single variable is called an ordinary differential equation.

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Examples:

$$\frac{dy}{dx} + 18x = 0$$

$$\frac{d^2y}{dx^2} - x\sqrt{y}\frac{dy}{dx} = y - 2$$

A differential equation involving a function of several variables is called a partial differential equation.

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Examples:

$$\frac{\partial^2 z}{\partial x^2} + 2x = 0$$

$$\frac{\partial z}{\partial y} = \sin x \frac{\partial z}{\partial x} + \cos y$$

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Partial differential equations are even harder.

We'll focus only on ordinary differential equations.

The highest order derivative in a differential equation defines the order of the equation.

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Below is an equation of order 1 and an equation of order 2.

$$\frac{dy}{dx} = y \qquad \text{order 1}$$

$$\frac{d^2y}{dx^2} = y \qquad \text{order 2}$$

If we are given an nth order differential equation along with the values, at a single input, of the function and its first n-1 derivatives, then we call that an initial value problem.

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If we are given an nth order differential equation along with the values, at several inputs, of the function and its first n-1 derivatives, then we call that a boundary value problem.

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