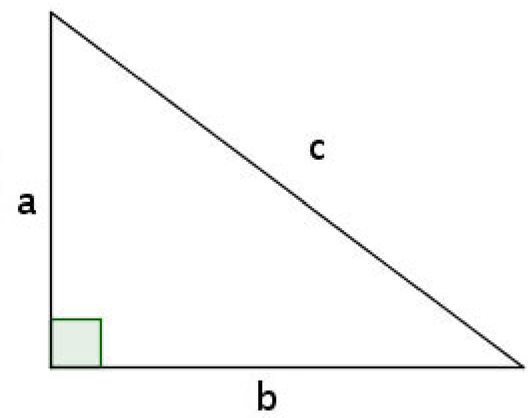
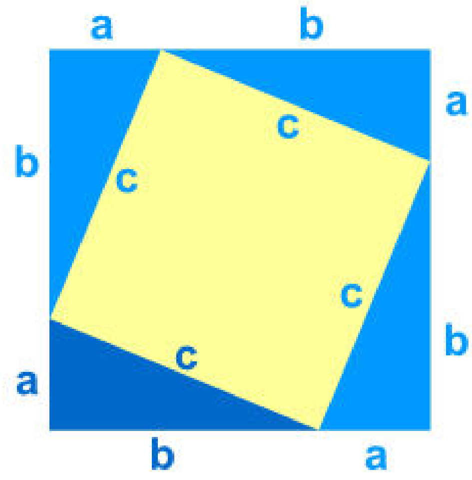
THE PYTHAGOREAN THEOREM

Theorem: Suppose you have a right triangle such as the one below with legs that have length *a* and *b*, and the hypotenuse has length *c*. Then it is always true that $a^2 + b^2 = c^2$.



Proof: There exist lots of different proofs of the Pythagorean Theorem. In fact, President James Garfield once published his own original proof of the theorem. However, the proof we will give here is both very easy and also the one that is probably seen most often. We begin with the following diagram that is composed of four identical right triangles that surround a square at the center.



We'll now find the area of the entire object (the big square) in two different ways. On the one hand, we have a square such that the length of each side is $a+b$. Thus, the area of the big square is $(a+b)(a+b) = a^2 + 2ab + b^2$. But on the other hand, the area of the big square is also equal to the sum of the areas of its component parts, namely the smaller square at the center plus the areas of the four identical right triangles. In particular, the area of the yellow square at the center is c^2, and the area of each one of the four identical right triangles is $\frac{ab}{2}$. Thus, the area of the entire object, the big square, is $c^2 + 4\left(\frac{ab}{2}\right) = c^2 + 2ab$. Hence, if the area of the big square is both $c^2 + 2ab$ and $a^2 + 2ab + b^2$, then we must have that $a^2 + 2ab + b^2 = c^2 + 2ab$. Now just subtract $2ab$ from both sides, and we're left with $a^2 + b^2 = c^2$. And that's it!

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