SUBGROUP OF A GROUP – ANSWER

<u>Theorem:</u> Let G be a group and let H be a subset of G. If for every $a \in H$ we have that $a^{-1} \in H$ and if for every $a, b \in H$ we have that $ab \in H$, then H is a subgroup of G.

<u>Proof:</u> Let G be a group and let H be a subset of G, and assume that for every $a \in H$ we have that $a^{-1} \in H$ and for every $a, b \in H$ we have that $ab \in H$. To show that H is a subgroup of G, we need to show four things – closure under the group multiplication, the associative law, the existence of an identity, and the existence of inverses. We are assuming in our hypothesis that the closure and inverse properties are satisfied, and we get the associative property for free since it holds for all elements in the group G. Thus, we just need to establish the existence of an identity element. But this is easy because if $a \in H$, then $a^{-1} \in H$, and since we are assuming closure under multiplication in H, we have that $aa^{-1} = e \in H$. Therefore, H is a subgroup of G.

$\square$