

A CONDITION FOR BEING AN ABELIAN GROUP – ANSWER

Theorem: Let G be a group. If $x^2 = e$ for every $x \in G$, then G is abelian.

Proof: Let G be a group, let $a, b \in G$, and suppose that for every $x \in G$, $x^2 = e$. Then, in particular, $(ab)^2 = (ab)(ab) = e$, the identity. Hence, $ab = (ab)^{-1} = b^{-1}a^{-1}$. But since we also have that $a^2 = aa = e$ and $b^2 = bb = e$, it follows that $a = a^{-1}$ and $b = b^{-1}$. Therefore, $ab = (ab)^{-1} = b^{-1}a^{-1} = ba$ and G is abelian.

□