THE STABILIZER SUBGROUP – ANSWER

<u>Definition:</u> Let *G* be a group that acts upon a set *X*, and let $x \in X$. Then the <u>stabilizer of x by *G*</u> is $Stabilizer_G(x) = G_x = \{g \in G \mid g(x) = x\}$.

<u>Theorem:</u> If *G* acts on a set *X* and if $x \in X$, then the stabilizer of *G* on x is a subgroup of *G*.

<u>Proof:</u> Let *G* be a group that acts on a set *X*, let $x \in X$, and let $Stabilizer_G(x) = G_x = \{g \in G \mid g(x) = x\}$. To show that G_x is a subset of *G*, we just need to show closure under composition of functions and existence of inverses for functions in G_x. Thus, suppose that $g_1, g_2 \in G_x$. Then $(g_1 \circ g_2)x = g_1(g_2(x)) = g_1(x) = x \Rightarrow g_1 \circ g_2 \in G_x$. Hence, closure is satisfied. Also, if $g(x) = x$ for $g \in G_x$, then $x = g^{-1}(x) \Rightarrow g^{-1} \in G_x$. Therefore, G_x is a subgroup of *G*.

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