SUBGROUP GENERATED BY CONJUGATES – ANSWER

Theorem: Given a group *G*, a subgroup *H*, and a set *M* equal to all the subgroups conjugate to *H*, then the subgroup generated by elements of *M* is a normal subgroup of *G*.

Proof: Notice that for any group *G* and subgroup *H*, *M* is not empty since $H \subseteq M$. To illustrate, by way of example, the essence of the proof, suppose that $M = \{H, H_2, H_3\}$. In other words, these three subgroups and only these three subgroups are conjugate to *H* in *G*. Then the subgroup generated by *M*, denoted by $\langle M \rangle$, consists of all finite products of elements in *H*, H_2, and/or H_3. Thus, suppose *abc* is one such finite product and that $g \in G$. To show that $\langle M \rangle$ is normal in *G*, it suffices to show that the conjugate of an element like *abc* by *g* is still an element of $\langle M \rangle$. But again, this is easy because $g^{-1}(abc)g = g^{-1}ag \cdot g^{-1}bg \cdot g^{-1}cg$. Now since any conjugate of *a*, *b*, or *c* has to be an element of *H*, H_2, or H_3, it follows that the product of these conjugates is an element of $\langle M \rangle$. Therefore, the subgroup generated by elements of *M* is a normal subgroup of *G*.

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