CAYLEY'S THEOREM REVISITED – ANSWER

Theorem: Every group *G* is isomorphic to a group of permutations acting on a set of objects. (2nd Proof)

Proof: Here we will give a very different proof of Cayley's Theorem that every group is isomorphic to a group of permutations. Notice, by the way, that we have dropped the finite condition. The crux of this proof is that if $g \in G$, then the function $T_g : G \to G$ defined by $T_g(x) = gxg^{-1}$ is, by previous proof, a bijection from *G* onto *G*, and that also means that it creates a permutation of the elements of *G*. Furthermore, we can identify each permutation created with the corresponding function of the form $T_g(x) = gxg^{-1}$. To show that the collection of all such functions/permutations along with the operation of function composition is a group that is isomorphic to *G*, all that remains to be shown is that the results of multiplication are preserved, In other words, show that if $a, b, c \in G$ such that $ab = c$, then $T_a \circ T_b = T_{ab} = T_c$. But this is easy. For example, if $x \in G$, then $(T_a \circ T_b)(x) = T_a(T_b(x)) = T_a(bxb^{-1}) = abxb^{-1}a^{-1} = abx(ab)^{-1} = T_{ab}(x) = T_c(x)$. Therefore, every group *G* is isomorphic to a group of permutations acting on a set of objects.

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