AN IMPORTANT ISOMORPHISM – ANSWER

Theorem: Let G be a group, let $g \in G$, and define a function $T_g : G \to G$ by $T_g(x) = gxg^{-1}$. Then T_g is an isomorphism.

Proof: To show that T_g is an isomorphism, we need to show first that it is a bijection and second that it preserves the multiplication. In other words, show that if $ab = c$, the $T_g(a) \cdot T_g(b) = T_g(c) = T_g(ab)$. In the last theorem we showed that T_g is a bijection, so let's show the second part. Thus, suppose $ab = c$ for $a, b, c \in G$, and suppose that $g \in G$. Then $T_g(a) \cdot T_g(b) = gag^{-1} \cdot gbg^{-1} = g(aeb)g^{-1} = g(ab)g^{-1} = gcg^{-1} = T_g(c) = T_g(ab)$. Therefore, T_g is an isomorphism.

□