AN IMPORTANT BIJECTION – ANSWER

<u>Theorem:</u> Let G be a group, let $g \in G$, and define a function $T_g : G \to G$ by $T_g(x) = gxg^{-1}$. Then $T_g : G \to G$ is a one-to-one and onto function, or in other words, a bijection.

<u>Proof:</u> Let $T_g : G \to G$ be defined by $T_g(x) = gxg^{-1}$ for $x \in G$. To show that T_g is one-to-one, we just need to demonstrate that if $T_g(x) = T_g(y)$, then $x = y$. However, this follows immediately from our right and left cancellation laws in a group. That is, $T_g(x) = T_g(y) \Rightarrow gxg^{-1} = gyg^{-1} \Rightarrow g^{-1}(gxg^{-1})g = g^{-1}(gyg^{-1})g \Rightarrow exe = eye \Rightarrow x = y$. Thus, T_g is one-to-one.

To show that T_g is onto, that means that if $b \in G$, then there exists $x \in G$ such that $T_g(x) = b$. But it is easy to find such an *x*. Just let $x = g^{-1}bg$. Then $T_g(x) = T_g(g^{-1}bg) = g(g^{-1}bg)g^{-1} = ebe = b$, and T_g is onto. Therefore, $T_g : G \to G$ be defined by $T_g(x) = gxg^{-1}$ for $x \in G$ is a bijection.

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