MORE CORRESPONDENCES – ANSWER

Theorem: Let G be a group, N a normal subgroup of G, and let M be a subgroup of G/N that contains N. Also, define $f: G \to G/N$ by $f(g) = Ng$, and define $f^{-1}: G/N \to G$ by $f^{-1}(Ng) = \{g \in G \mid f(g) \in Ng\}$. Similarly, for any set $A \subseteq G/N$ let $f^{-1}(A) = \{g \in G \mid f(g) \in A\}$. Then if M is a normal subgroup of G/N, $f^{-1}(M)$ is a normal subgroup of G.

Proof: We have already established that $f^{-1}(M)$ is a subgroup of G, so all we need to do is establish that the normality of $f^{-1}(M)$ does indeed follow from the normality of M in G/N. To do this, observe that if $g \in G$ and $h \in f^{-1}(M)$, then $(Ng)^{-1} \cdot Nh \cdot Ng = Ng^{-1} \cdot Nh \cdot Ng = N(g^{-1}hg) \in M$. But this also means that $g^{-1}hg \in f^{-1}(M)$. Hence, if $g \in G$ and $h \in f^{-1}(M)$, then $g^{-1}hg \in f^{-1}(M)$, and therefore, $f^{-1}(M)$ is a normal subgroup of G.

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