CORRESPONDENCE OF NORMAL SUBGROUPS – ANSWER

Theorem: If H is a normal subgroup of a group G and if N is a normal subgroup of G, then the right (left) cosets corresponding to elements of H form a normal subgroup of G/N.

Proof: In our previous theorem we demonstrated that the right (left) cosets corresponding to elements of H form a subgroup of G/N, and so all that is left is to demonstrate that this will be a normal subgroup of G/N. Thus, note that since H is normal in G, if $g \in G$ and $h \in H$, then $g^{-1}hg \in H$. Consequently, if also follows that $(Ng)^{-1} \cdot Nh \cdot Ng = Ng^{-1} \cdot Nh \cdot Ng = N(g^{-1}hg)$ where, again, $g^{-1}hg \in H$. Therefore, the cosets in G/N corresponding to elements of H form a normal subgroup of G/N.

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