CORRESPONDENCE OF SUBGROUPS – ANSWER

Theorem: If H is a subgroup of a group G and if N is a normal subgroup of G, then the right (left) cosets corresponding to elements of H form a subgroup of G/N.

Proof: Let H be a subgroup of G, let N be a normal subgroup of G, and consider the right (left) cosets in G/N that correspond to elements of H. By previous proof, we know that when N is a normal subgroup of G that multiplication in G/N defined by $Na \cdot Nb = N(ab)$ is well-defined, and recall that that means that it doesn't matter which elements of the cosets we use when performing the multiplication. Thus, to show that the cosets corresponding to elements in H form a subgroup, all we need to do is demonstrate closure under multiplication and the existence of inverses. But that is easy to do. For example, if $h_1, h_2 \in H$, then $Nh_1 \cdot Nh_2 = N(h_1h_2)$ is also a right coset involving an element of H since $h_1h_2 \in H$. Similarly, if $h, h^{-1} \in H$, then $Hh \cdot Hh^{-1} = H(hh^{-1}) = He = H$ implies that $Hh^{-1} = (Hh)^{-1}$. Hence, inverses also exist in this collection of cosets, and so the cosets in G/N that correspond to elements of H form a subgroup of G/N.

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