COMMUTATIVITY IN NORMAL SUBGROUPS – ANSWER

Theorem: Let G be a group, let M and N be normal subgroups of G such that $M \cap N = e$ (the identity), and let $m \in M$ and $n \in N$. Then m and n commute with one another, or in other words, $mn = nm$.

Proof: Let G be a group, let M and N be normal subgroups of G such that $M \cap N = e$ (the identity), and let $m \in M$ and $n \in N$. Then by our previous proof, the commutator $m^{-1}n^{-1}mn$ is in the intersection of M and N, But this means that $m^{-1}n^{-1}mn = M \cap N = e$. However, $m^{-1}n^{-1}mn = e \Rightarrow n^{-1}mn = m \Rightarrow mn = nm$. Therefore, m and n commute with one another.

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