COMMUTATORS IN NORMAL SUBGROUPS – ANSWER

Theorem: Let G be a group, let M and N be normal subgroups of G, and let $m \in M$ and $n \in N$. Then the commutator of m by n, $m^{-1}n^{-1}mn$, is an element of $M \cap N$.

Proof: Let G be a group, let M and N be normal subgroups of G, and let $m \in M$ and $n \in N$, and consider the commutator $m^{-1}n^{-1}mn$. Since N is a normal subgroup of G, it follows that $m^{-1}n^{-1}m \in N$ and, hence, $(m^{-1}n^{-1}m)n = m^{-1}n^{-1}mn \in N$. On the other hand, since M is a normal subgroup of G, it also follows that $n^{-1}mn \in M$ and, hence, $m^{-1}(n^{-1}mn) = m^{-1}n^{-1}mn \in M$. Therefore, $m^{-1}n^{-1}mn \in M \cap N$.

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