GROUPS WITH AN EVEN NUMBER OF ELEMENTS – ANSWER

<u>Theorem:</u> If a finite group *G* has an even number of elements, then at least one non-identity element is its own inverse.

<u>Proof:</u> We will illustrate the argument by assuming we have a group of order 8. If we remove the identity element, then that leaves 7 non-identity elements. Now let's consider the consequences of each of the remaining elements having an inverse that is different from itself. If this were the case, then we would need an even number of elements every non-identity element would be paired with a different element that is also its inverse. However, since in actuality we have 7 non-identity elements in the group, it follows that at least one of the elements is its own inverse. And now, you, hopefully, see that this same argument can be applied to any finite group with an even number of elements.

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