THE SUBGROUP GENERATED BY CONJUGATE SUBGROUPS – ANSWER

Theorem: If H is a subgroup of a group G, then the subgroup N generated by H and its conjugates is normal in G.

Proof: Let G be a group and let H be a group. If H is normal (self-conjugate) in G, then set N equal to H and we are done. On the other hand, if G contains several subgroups that are conjugate to H, then let N be the subgroup generated by taking all finite products of elements in H and the corresponding conjugates of H. Now let abc represent a typical product of such elements and let $g \in G$, and let's consider the product $g^{-1}(abc)g$. Clearly, we could also write this as $g^{-1}(abc)g = g^{-1}a(gg^{-1})b(gg^{-1})cg) = (g^{-1}ag)(g^{-1}bg)(g^{-1}cg)$. From this last form we see that $g^{-1}(abc)g$ will be equal to a product of conjugates of a, b, and c, and, thus, $g^{-1}(abc)g$ belongs to the subgroup N generated by elements of H and its conjugates. Therefore, N is a normal subgroup of G.

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