THE EVEN SUBGROUP – ANSWER

Definition: A permutation is even if it can be written as a product of an even number of transpositions, i.e. cycles of length two. Also, the identity is always considered to be an even permutation.

Theorem: Let *G* be a group of permutations. Then the set of all even permutations in *G* form a normal subgroup.

Proof: Let *G* be a group of permutations. Then the set *E* of even permutations in *G* is non-empty since the identity is an even permutation. Now suppose $a,b \in E$. Then since *a* and *b* can be written as a product of an even number of transpositions, it immediately follows that *ab* is also the product of an even number of transpositions. In other words, if you follow the even number of transpositions for *a* by the even number of transpositions for *b*, then you will have an even number of transpositions whose product is *ab*. Furthermore, since we can construct a^{-1} by just writing the transpositions for *a* in reverse order, it follows that a^{-1} is also an even permutation. Thus, *E* is a subgroup of *G*.

Now let $a \in E$ and let $g \in G$, and let's consider $g^{-1}ag$. If *g* is an even permutation, then g^{-1} is also an even permutation, and it easily follows that $g^{-1}ag$ can be written as the product of an even number of transpositions (since even + even + even = even). Similarly, if *g* is an odd permutation, then g^{-1} is also an odd permutation. However, $g^{-1}ag$ is still an even permutation since this product written as a product of transpositions will still contain an even number of transpositions (odd + even + odd = even). Thus, for any $a \in E$ and any $g \in G$, we'll always have that $g^{-1}ag \in E$. Therefore, *E* is a normal subgroup of *G*.

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