THE COMMUTATOR SUBGROUP IS NORMAL – ANSWER

Definition: The commutator or derived subgroup of a group *G*, denoted by G', is the set of all finite products of commutators in *G* where a commutator is a product of either the form $a^{-1}b^{-1}ab$ or $aba^{-1}b^{-1}$.

Theorem: The commutator (or derived) subgroup of a group *G* is normal in *G*.

Proof: First of all, by definition the set of commutators is closed under multiplication. Also, if $a^{-1}b^{-1}ab$ is a commutator in *G*, then its inverse, $(a^{-1}b^{-1}ab)^{-1} = b^{-1}a^{-1}(b^{-1})^{-1}(a^{-1})^{-1} = b^{-1}a^{-1}ba$ is also a commutator in *G*. From this it follows that any finite product of commutators will have an inverse that belongs to the set of all finite products of commutators in *G*, and, hence, the set of all finite products of commutators in *G* is a subgroup of *G*. To show that the commutator subgroup is a normal subgroup of *G*, let $a \in G'$ and let $b \in G$. It now suffices to show that $b^{-1}ab \in G'$. To do this, note that $(b^{-1}ab)a^{-1} = b^{-1}aba^{-1}$ is the commutator of *b* and a^{-1}, and, hence, it is equal to some element *c* in G'. But if $(b^{-1}ab)a^{-1} = b^{-1}aba^{-1} = c \in G'$, then $b^{-1}ab = ca$. However, since $a, c \in G'$, that means that $b^{-1}ab = ca \in G'$, and that means that the commutator subgroup of a group *G* is normal in *G*.

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