QUOTIENT GROUPS – ANSWER

Theorem: If N is a normal subgroup of a group G, then $G/N = \{Na \mid a \in G\}$ is a group where the multiplication of cosets is defined in terms of the multiplication of elements in G. In other words, $Na \cdot Nb = N(ab)$.

Proof: In a previous proof we showed that this multiplication is well-defined. That means that we get the same result regardless of which element from a coset is used to represent it. Having noted that, it's obvious that the closure property holds. In other words, if $a, b \in G$, then the product of two right cosets is again a right coset, $Na \cdot Nb = N(ab)$. Furthermore, we get the associated property for free, because multiplication in G is associative. Hence, $N(ab)c = Na(bc)$. Additionally, the identity element in G/N is $N = Ne$. Furthermore, for any coset Na its inverse is Na^{-1} since $Na \cdot Na^{-1} = N(aa^{-1}) = Ne = N$. Therefore, G/N with the multiplication inherited from G is a group.

$\square$