COSETS AND EQUIVALENCE RELATIONS – PRACTICE

Definition: If X is a non-empty set, then a relation between elements in X is called an equivalence relation if and only if the following conditions are met:

1. For every $a \in X$, $a \equiv a$ (reflexive),
2. For every $a, b \in X$, if $a \equiv b$, then $b \equiv a$ (symmetric), and
3. For every $a, b, c \in X$, if $a \equiv b$ and $b \equiv c$, then $a \equiv c$ (transitive).

Theorem: If H is a subgroup of a group G, then the right (left) cosets of H in G define an equivalence relation.

Proof: