SIZE OF COSETS – ANSWER

Theorem: If H is a subgroup of a finite group G, then any two right (left) cosets have the same number of elements.

Proof: We will prove the theorem just for right cosets since the argument for left cosets is the same. Thus, let H is a subgroup of a finite group G and suppose that $a \in G$ and that H and Ha are distinct right cosets. Recall that if H has m elements, $e = h_1, h_2, h_3, \ldots, h_m$,then the members of Ha are $a, h_2 a, h_3 a, \ldots, h_m a$. It now follows from the right cancellation law that these are m distinct elements in Ha since otherwise if, for example, we had $h_2 a = h_3 a$, then this would incorrectly imply that $h_2 = h_3$. And since a was chosen to be any arbitrary element that is not in H, this argument shows that all right cosets of H in G will have the same number of elements as the subgroup H. Therefore, any two right cosets of H in G have the same number of elements.

□

NOTE: This proof can be extended to include infinite groups, but we don't want to get into the complexities of infinite sets at this point.