SUBGROUP OF A FINITE GROUP – ANSWER

<u>Theorem:</u> Let G be a finite group and let H be a subset of G. If for every $a, b \in H$ we have that $ab \in H$, then H is a subgroup of G.

<u>Proof:</u> Let G be a finite group and let H be a subset of G, and assume that for every $a, b \in H$ we have that $ab \in H$. Now let $a \in H$. Then our closure property tells us that all powers of a must also belong to H. But since G is a finite group, eventually one of our powers of a will have to be equal to the identity. More specifically, if the order of G is n, $|G| = n$, then because G has only a finite number of elements, at least one of the powers in the list $a, a^2, a^3, \ldots, a^n$ must be the identity. Thus, it follows from the closure property that not only is $e \in H$, but $a^{-1} \in H$ as well. Therefore, H is a subgroup of G.

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