THE UNIQUENESS OF THE IDENTITY – ANSWER

Theorem: A group G has a unique identity element. In other words, it has only one element e with the property that for every $a \in G$, $e \cdot a = a = a \cdot e$.

Proof: Suppose that e_1 and e_2 are both identity elements in G. Then since e_1 is an identity element, $e_1 \cdot (e_2) = e_2$. On the other hand, since e_2 is an identity element, $(e_1) \cdot e_2 = e_1$. Therefore, $e_1 = e_1 \cdot e_2 = e_2$, and the identity element in a group is unique.

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