THE NATURAL HOMOMORPHISM

Discussion: We've talked before about normal subgroups, such as when *N* is a normal subgroup of *G*, and the corresponding quotient groups, such as G/N. What we want to demonstrate now is that there is a very obvious surjective homomorphism from *G* onto G/N that we call the *natural homomorphism*.

Theorem: Let *G* be a group, and let *N* be a normal subgroup of *G*. Then the function $\pi : G \to G/N$ defined by $\pi(g) = Ng$ is a homomorphism from *G* onto G/N. This homomorphism is called the *natural homomorphism*.

Proof: By previous proof (see Theorem 19), we know that the right (left) cosets of *N* in *G* form a group under the multiplication inherited from *G*. We also know that the function we've defined is onto since if $Ng \in G/N$, then $Ng = \pi(g)$ for $g \in G$. To show that $\pi : G \to G/N$ is a homomorphism, let $a, b \in G$. Then $\pi(ab) = Nab = NaNb = \pi(a)\pi(b)$.

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