

## Lesson 18

### THE FUNDAMENTAL THEOREM OF FINITE ABELIAN GROUPS – ANSWERS

Find, in terms of direct products of cyclic groups of prime power order, all finite abelian groups of the given order.

1. Order 4

two,  $\mathbb{Z}_4$   
 $\mathbb{Z}_2 \times \mathbb{Z}_2$

2. Order 6

one,  $\mathbb{Z}_6 \cong \mathbb{Z}_2 \times \mathbb{Z}_3$

3. Order 7

one,  $\mathbb{Z}_7$

4. Order 8

three,  $\mathbb{Z}_8$   
 $\mathbb{Z}_2 \times \mathbb{Z}_4$   
 $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$

5. Order 12

two,  $\mathbb{Z}_{12} \cong \mathbb{Z}_3 \times \mathbb{Z}_4$   
 $\mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2$

6. Order 16

five,  $\mathbb{Z}_{16}$   
 $\mathbb{Z}_2 \times \mathbb{Z}_8$   
 $\mathbb{Z}_4 \times \mathbb{Z}_4$ ,  
 $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_4$ ,  
 $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$