

## Lesson 4

### SYMMETRIC GROUPS – PRACTICE

1. Find the number of elements in  $S_5, S_6$ , and  $S_7$ .

$$|S_5| = 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

$$|S_6| = 6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$$

$$|S_7| = 7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$$

2. Find the number of elements in  $A_5, A_6$ , and  $A_7$ .

$$|A_5| = \frac{|S_5|}{2} = \frac{120}{2} = 60$$

$$|A_6| = \frac{|S_6|}{2} = \frac{720}{2} = 360$$

$$|A_7| = \frac{|S_7|}{2} = \frac{5040}{2} = 2520$$

3. Find  $\frac{|S_5|}{|A_5|}$ ,  $\frac{|S_6|}{|A_6|}$ , and  $\frac{|S_7|}{|A_7|}$ .

$$\frac{|S_5|}{|A_5|} = 2$$

$$\frac{|S_6|}{|A_6|} = 2$$

$$\frac{|S_7|}{|A_7|} = 2$$

4. Below is a list of the 24 elements in  $S_4$ . Find the 12 elements in  $A_4$ .

$$S_4 = \{ (), (3,4), (2,3), (2,3,4), (2,4,3), (2,4), (1,2), (1,2)(3,4), (1,2,3), (1,2,3,4), (1,2,4,3), (1,2,4), (1,3,2), (1,3,4,2), (1,3), (1,3,4), (1,3)(2,4), (1,3,2,4), (1,4,3,2), (1,4,2), (1,4,3), (1,4), (1,4,2,3), (1,4)(2,3) \}$$

$$A_4 = \{ (), (2,3,4), (2,4,3), (1,2)(3,4), (1,2,3), (1,2,4), (1,3,2), (1,3,4), (1,3)(2,4), (1,4,2), (1,4,3), (1,4)(2,3) \}$$

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5. First find the 8 elements in  $D_4$ , and then find  $Even(D_4)$ .

$$D_4 = \{ (), (2,4), (1,2)(3,4), (1,2,3,4), (1,3), (1,3)(2,4), (1,4,3,2), (1,4)(2,3) \}$$

$$Even(D_4) = \{ (), (1,2)(3,4), (1,3)(2,4), (1,4)(2,3) \}$$