SYMBOLIC LOGIC – PRACTICE

This lesson is an introduction to symbolic logic and what we actually mean in mathematics by statements such as "*a* implies *b*" and "*a* if and only if *b*." Below are some common symbols that are used in logic followed by the corresponding math symbols that I will use instead.

LOGIC	**MATH**
~ or $\neg$	not
$\vee$	or
$\wedge$	and
$a \to b$	$a \Rightarrow b$
$a \leftrightarrow b$	$a \Leftrightarrow b$

The statement "$a \Rightarrow b$" can be read as *"a implies b"* or *"if a then b"* or *"a is a sufficient condition for b"* or *"b is a necessary condition for a."*

The statement "$a \Leftrightarrow b$" can be read or written as *"a iff b"* or *"a if and only if b"* or *"a is a necessary and sufficient condition for b."*

Using the logical connectives above, we can rewrite "$a \Rightarrow b$" as "not (a & not-b) ." Similarly, since "$a \Leftrightarrow b$" means "$a \Rightarrow b$ & $b \Rightarrow a$," we can rewrite "$a \Leftrightarrow b$" as "[not (a & not-b)] & [not (b & not-a)]."

In mathematics, for the compound statement *"A & B"* to be true, both of the statements *A* and *B* must be true. On the other hand, for the compound statement *"A or B"* to be true, only one of the statements must be true. Construct some simple examples to convince yourself that this is the correct way to proceed. Also, in mathematics, unless stated otherwise, we always use an *inclusive or*. That means that for *"A or B"* to be true, we either have *A* true or *B* true or both *A* and *B* true. In an *exclusive or*, either *A* or *B* can be true, but not both at the same time.

Now, here are some things for you to either do or look up and respond to.

1. What is a *truth table?*

2. What is the *law of the excluded middle?*

3. What is a *tautology?*

4. What is a *contradiction?*

5. Complete the following *truth tables*. Your final values should be in the column shaded yellow.

Not	**A**
	T
	F

A	**&**	**B**

A	**or**	**B**

Not	[A	&	(Not	B)]

{ Not	[A	&	(Not	B)] }	&	{ Not	[B	&	(Not	A)] }

6. What is *modus ponens?* Give an example.

7. What is *modus tollens"* Give an example.

8. Explain why a false statement can imply anything? In other words, why is a statement such as *"If the moon is made of green cheese, then I am the smartest man on the planet"* considered to be true?

9. Use truth tables to show that " not (a & not-b) " and "not-a or b" are *logically equivalent*. In this context, logical equivalence means that if a and b are assigned the same truth values, then the truth values of our final compound statements are the same.

10. Consider the statement *"This sentence is false."* What are the implications of this statement being true? What are the implications of it being false? Read http://en.wikipedia.org/wiki/Paradox.

11. What is the *inverse* of $a \Rightarrow b$?
    What is the *converse* of $a \Rightarrow b$?
    What is the *contrapositive* of $a \Rightarrow b$?
    Use truth tables to show that $a \Rightarrow b$ is *logically equivalent* to $\sim b \Rightarrow \sim a$ (not-b implies not-a).

12. There is another logical operator worth mentioning that is known as NAND or the Sheffer stroke. NAND stands for "not and," and it can be defined as " not (a & b) ." Also, *"a NAND b"* is generally written as " $a \uparrow b$." Use truth tables to show that $a \uparrow b$ is logically equivalent to "not-a or not-b."