

Lesson 9

SUBGROUP LATTICES – PRACTICE

Below you will find the five groups of order 8 along with a set of subgroups. Construct the subgroup lattice for each.

$$1. H_1 = C_8 = \{ (), (1,2,3,4,5,6,7,8), (1,3,5,7)(2,4,6,8), (1,4,7,2,5,8,3,6), (1,5)(2,6)(3,7)(4,8), (1,6,3,8,5,2,7,4), (1,7,5,3)(2,8,6,4), (1,8,7,6,5,4,3,2) \}$$

$$H_2 = \{ (), (1,5)(2,6)(3,7)(4,8), (1,3,5,7)(2,4,6,8), (1,7,5,3)(2,8,6,4) \}$$

$$H_3 = \{ (), (1,5)(2,6)(3,7)(4,8) \}$$

$$H_4 = \{ () \}$$

$$2. H_1 = D_4 = \{ (), (2,4), (1,2)(3,4), (1,2,3,4), (1,3), (1,3)(2,4), (1,4,3,2), (1,4)(2,3) \}$$

$$H_2 = \{ (), (1,3), (2,4), (1,3)(2,4) \}$$

$$H_3 = \{ (), (1,3)(2,4), (1,2)(3,4), (1,4)(2,3) \}$$

$$H_4 = \{ (), (1,2,3,4), (1,3)(2,4), (1,4,3,2) \}$$

$$H_5 = \{ (), (1,3) \}$$

$$H_6 = \{ (), (1,4)(2,3) \}$$

$$H_7 = \{ (), (2,4) \}$$

$$H_8 = \{ (), (1,2)(3,4) \}$$

$$H_9 = \{ (), (1,3)(2,4) \}$$

$$H_{10} = \{ () \}$$

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3. $H_1 = C_2 \times C_4 = \{ (), (3,4,5,6), (3,5)(4,6), (3,6,5,4), (1,2), (1,2)(3,4,5,6), (1,2)(3,5)(4,6), (1,2)(3,6,5,4) \}$

$$H_2 = \{ (), (3,5)(4,6), (1,2)(3,4,5,6), (1,2)(3,6,5,4) \}$$

$$H_3 = \{ (), (3,4,5,6), (3,5)(4,6), (3,6,5,4) \}$$

$$H_4 = \{ (), (3,5)(4,6), (1,2), (1,2)(3,5)(4,6) \}$$

$$H_5 = \{ (), (1,2)(3,5)(4,6) \}$$

$$H_6 = \{ (), (3,5)(4,6) \}$$

$$H_7 = \{ (), (1,2) \}$$

$$H_8 = \{ () \}$$

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$$4. H_1 = C_2 \times C_2 \times C_2 = \{ (), (5,6), (3,4), (3,4)(5,6), (1,2), (1,2)(5,6), (1,2)(3,4), (1,2)(3,4)(5,6) \}$$

$$H_2 = \{ (), (3,4)(5,6), (1,2)(5,6), (1,2)(3,4) \}$$

$$H_3 = \{ (), (5,6), (1,2)(3,4), (1,2)(3,4)(5,6) \}$$

$$H_4 = \{ (), (3,4), (1,2)(5,6), (1,2)(3,4)(5,6) \}$$

$$H_5 = \{ (), (3,4)(5,6), (1,2), (1,2)(3,4)(5,6) \}$$

$$H_6 = \{ (), (5,6), (1,2), (1,2)(5,6) \}$$

$$H_7 = \{ (), (3,4), (1,2), (1,2)(3,4) \}$$

$$H_8 = \{ (), (3,4), (5,6), (3,4)(5,6) \}$$

$$H_9 = \{ (), (1,2)(3,4)(5,6) \}$$

$$H_{10} = \{ (), (1,2)(3,4) \}$$

$$H_{11} = \{ (), (1,2)(5,6) \}$$

$$H_{12} = \{ (), (1,2) \}$$

$$H_{13} = \{ (), (3,4)(5,6) \}$$

$$H_{14} = \{ (), (3,4) \}$$

$$H_{15} = \{ (), (5,6) \}$$

$$H_{16} = \{ () \}$$

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$$5. H_1 = Q = \{ (), (1,2,5,6)(3,8,7,4), (1,3,5,7)(2,4,6,8), (1,4,5,8)(2,7,6,3), \\ (1,5)(2,6)(3,7)(4,8), (1,6,5,2)(3,4,7,8), (1,7,5,3)(2,8,6,4), (1,8,5,4)(2,3,6,7) \}$$

$$H_2 = \{ (), (1,3,5,7)(2,4,6,8), (1,5)(2,6)(3,7)(4,8), (1,7,5,3)(2,8,6,4) \}$$

$$H_3 = \{ (), (1,2,5,6)(3,8,7,4), (1,5)(2,6)(3,7)(4,8), (1,6,5,2)(3,4,7,8) \}$$

$$H_4 = \{ (), (1,4,5,8)(2,7,6,3), (1,5)(2,6)(3,7)(4,8), (1,8,5,4)(2,3,6,7) \}$$

$$H_5 = \{ (), (1,5)(2,6)(3,7)(4,8) \}$$

$$H_6 = \{ () \}$$