

Lesson 9

SUBGROUP LATTICES – ANSWERS

Below you will find the five groups of order 8 along with a set of subgroups. Construct the subgroup lattice for each.

$$1. \quad H_1 = C_8 = \{ (), (1,2,3,4,5,6,7,8), (1,3,5,7)(2,4,6,8), (1,4,7,2,5,8,3,6), \\ (1,5)(2,6)(3,7)(4,8), (1,6,3,8,5,2,7,4), (1,7,5,3)(2,8,6,4), (1,8,7,6,5,4,3,2) \}$$

$$H_2 = \{ (), (1,5)(2,6)(3,7)(4,8), (1,3,5,7)(2,4,6,8), (1,7,5,3)(2,8,6,4) \}$$

$$H_3 = \{ (), (1,5)(2,6)(3,7)(4,8) \}$$

$$H_4 = \{ () \}$$

$$\left\{ \begin{array}{l} (), (1,2,3,4,5,6,7,8), (1,3,5,7)(2,4,6,8), (1,4,7,2,5,8,3,6), (1,5)(2,6)(3,7)(4,8), \\ (1,6,3,8,5,2,7,4), (1,7,5,3)(2,8,6,4), (1,8,7,6,5,4,3,2) \end{array} \right\}$$

$$\begin{array}{c} | \\ \left\{ \begin{array}{c} () \\ (1,5)(2,6)(3,7)(4,8), \\ (1,3,5,7)(2,4,6,8) \\ (1,7,5,3)(2,8,6,4) \end{array} \right\} \end{array}$$

$$\begin{array}{c} | \\ \left\{ \begin{array}{c} () \\ (1,5)(2,6)(3,7)(4,8) \end{array} \right\} \end{array}$$

$$\begin{array}{c} | \\ \{ () \} \end{array}$$

Lesson 9

2. $H_1 = D_4 = \{ (), (2,4), (1,2)(3,4), (1,2,3,4), (1,3), (1,3)(2,4), (1,4,3,2), (1,4)(2,3) \}$

$$H_2 = \{ (), (1,3), (2,4), (1,3)(2,4) \}$$

$$H_3 = \{ (), (1,3)(2,4), (1,2)(3,4), (1,4)(2,3) \}$$

$$H_4 = \{ (), (1,2,3,4), (1,3)(2,4), (1,4,3,2) \}$$

$$H_5 = \{ (), (1,3) \}$$

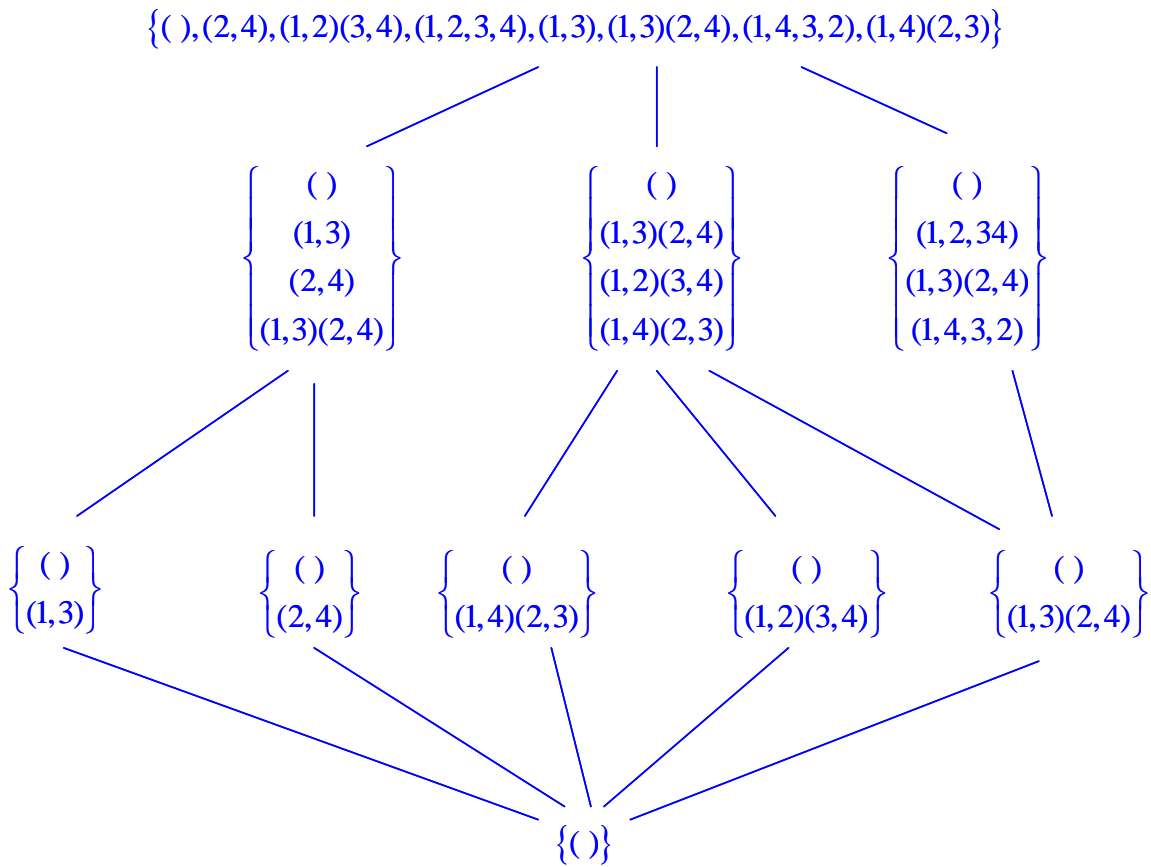
$$H_6 = \{ (), (1,4)(2,3) \}$$

$$H_7 = \{ (), (2,4) \}$$

$$H_8 = \{ (), (1,2)(3,4) \}$$

$$H_9 = \{ (), (1,3)(2,4) \}$$

$$H_{10} = \{ () \}$$



Lesson 9

$$3. \quad H_1 = C_2 \times C_4 = \{ (), (3,4,5,6), (3,5)(4,6), (3,6,5,4), (1,2), (1,2)(3,4,5,6), (1,2)(3,5)(4,6), (1,2)(3,6,5,4) \}$$

$$H_2 = \{ (), (3,5)(4,6), (1,2)(3,4,5,6), (1,2)(3,6,5,4) \}$$

$$H_3 = \{ (), (3,4,5,6), (3,5)(4,6), (3,6,5,4) \}$$

$$H_4 = \{ (), (3,5)(4,6), (1,2), (1,2)(3,5)(4,6) \}$$

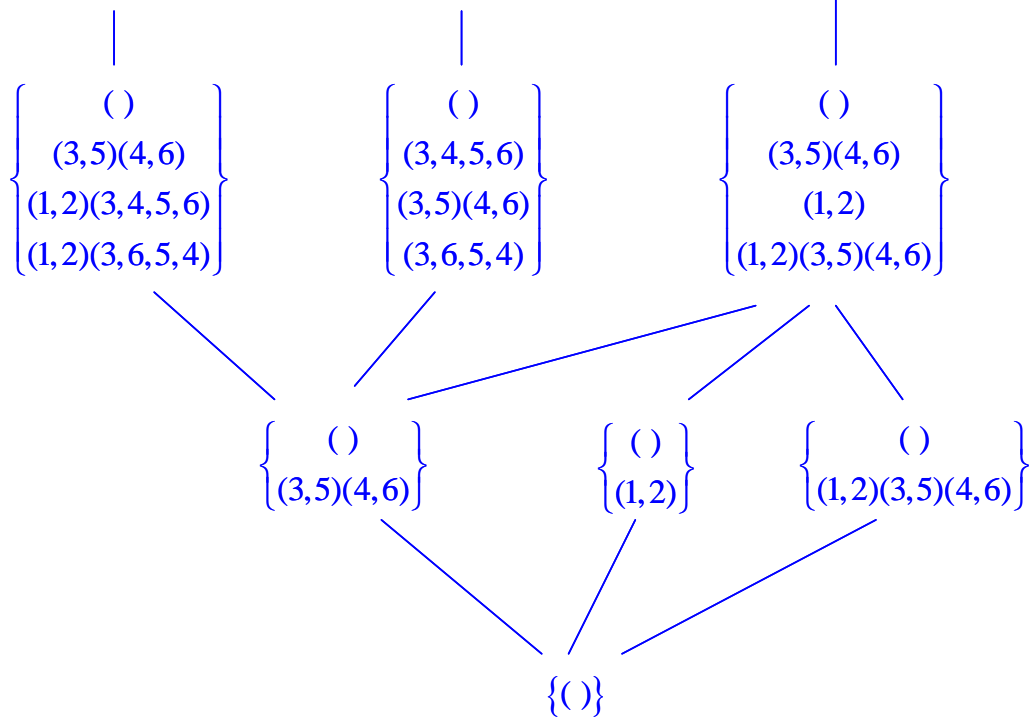
$$H_5 = \{ (), (1,2)(3,5)(4,6) \}$$

$$H_6 = \{ (), (3,5)(4,6) \}$$

$$H_7 = \{ (), (1,2) \}$$

$$H_8 = \{ () \}$$

$$\{ (), (3,4,5,6), (3,5)(4,6), (3,6,5,4), (1,2), (1,2)(3,4,5,6), (1,2)(3,5)(4,6), (1,2)(3,6,5,4) \}$$



Lesson 9

$$4. H_1 = C_2 \times C_2 \times C_2 = \{ (), (5,6), (3,4), (3,4)(5,6), (1,2), (1,2)(5,6), (1,2)(3,4), (1,2)(3,4)(5,6) \}$$

$$H_2 = \{ (), (3,4)(5,6), (1,2)(5,6), (1,2)(3,4) \}$$

$$H_3 = \{ (), (5,6), (1,2)(3,4), (1,2)(3,4)(5,6) \}$$

$$H_4 = \{ (), (3,4), (1,2)(5,6), (1,2)(3,4)(5,6) \}$$

$$H_5 = \{ (), (3,4)(5,6), (1,2), (1,2)(3,4)(5,6) \}$$

$$H_6 = \{ (), (5,6), (1,2), (1,2)(5,6) \}$$

$$H_7 = \{ (), (3,4), (1,2), (1,2)(3,4) \}$$

$$H_8 = \{ (), (3,4), (5,6), (3,4)(5,6) \}$$

$$H_9 = \{ (), (1,2)(3,4)(5,6) \}$$

$$H_{10} = \{ (), (1,2)(3,4) \}$$

$$H_{11} = \{ (), (1,2)(5,6) \}$$

$$H_{12} = \{ (), (1,2) \}$$

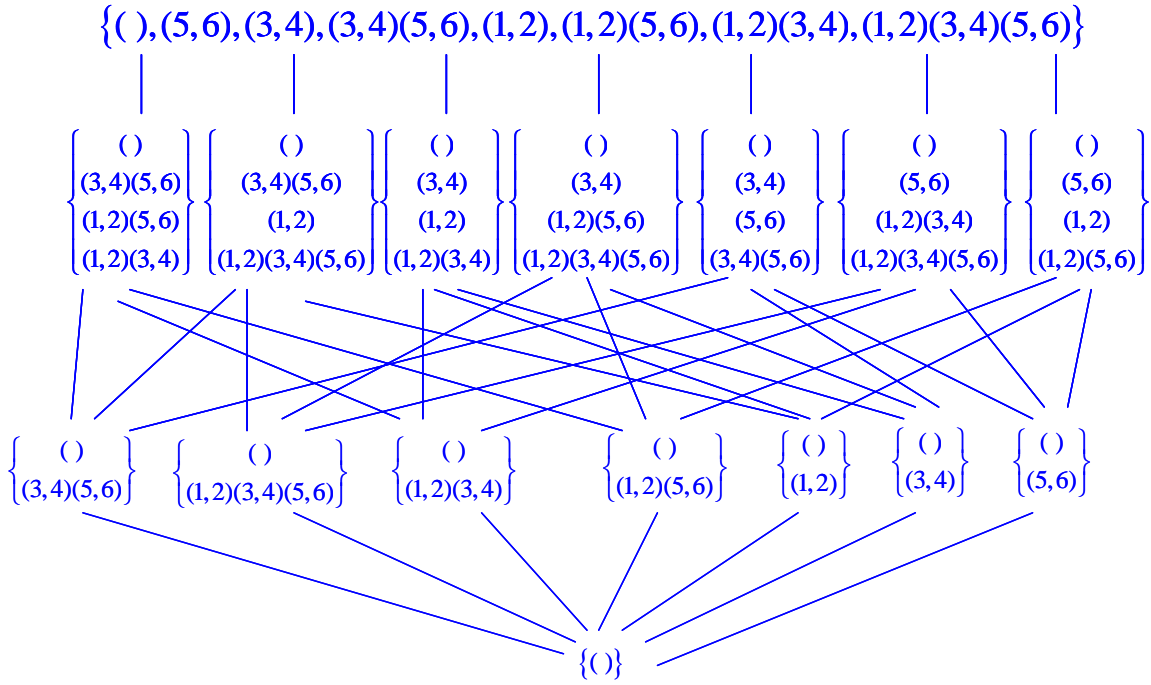
$$H_{13} = \{ (), (3,4)(5,6) \}$$

$$H_{14} = \{ (), (3,4) \}$$

$$H_{15} = \{ (), (5,6) \}$$

$$H_{16} = \{ () \}$$

Lesson 9



Lesson 9

$$5. H_1 = Q = \{ (), (1,2,5,6)(3,8,7,4), (1,3,5,7)(2,4,6,8), (1,4,5,8)(2,7,6,3), \\ (1,5)(2,6)(3,7)(4,8), (1,6,5,2)(3,4,7,8), (1,7,5,3)(2,8,6,4), (1,8,5,4)(2,3,6,7) \}$$

$$H_2 = \{ (), (1,3,5,7)(2,4,6,8), (1,5)(2,6)(3,7)(4,8), (1,7,5,3)(2,8,6,4) \}$$

$$H_3 = \{ (), (1,2,5,6)(3,8,7,4), (1,5)(2,6)(3,7)(4,8), (1,6,5,2)(3,4,7,8) \}$$

$$H_4 = \{ (), (1,4,5,8)(2,7,6,3), (1,5)(2,6)(3,7)(4,8), (1,8,5,4)(2,3,6,7) \}$$

$$H_5 = \{ (), (1,5)(2,6)(3,7)(4,8) \}$$

$$H_6 = \{ () \}$$

