Lesson 25

SMALL GROUPS

One of the best ways to get hands on experience with group theory is to study the structures of groups of small order, and one way to get information about these small groups is to Google "small groups wikipedia." This will take you to the informative article "List of Small Groups." Another way to get a lot of instant, valuable information about groups of small order is to download the free program "Group Explorer" by Nathan Carter. The homepage for this program is http://groupexplorer.sourceforge.net/, and this program was designed to accompany an equally excellent book by Nathan Carter titled "Visual Group Theory." And, of course, another invaluable aid for exploring small groups is the GAP software program. With this program you can easily identify subgroups, normal subgroups, conjugate subgroups, the center, and the commutator (derived) subgroup along with many other interesting properties of particular groups. Nonetheless, no matter how you do it, the more time you spend studying small groups, the more concrete group theory will become for you. It's worth the effort!

The rest of this lesson consists of examples I've put together for you on some of the critical properties of six groups of small order. In particular, the Klein 4-group, the dihedral group of degree 3, the quaternion group, the semidirect product of $\mathbb{Z}_3$ with $\mathbb{Z}_4$, the alternating group of degree 4, and the cyclic group of order 12. In each of the examples I've put together I've included crucial information such as generators for the group, generator diagrams, whether the group is abelian, and a list of all the subgroups in a way that identifies normal subgroups, conjugate subgroups, the center of the group, the commutator (derived) subgroup, and the even subgroup. These are crucial elements for helping you understand the internal structure of any group. The rest, however, is now up to you. You've been given some tools and some theory, and with these tools you can explore a vast universe. You can travel as far as you are willing to let your mind take you!