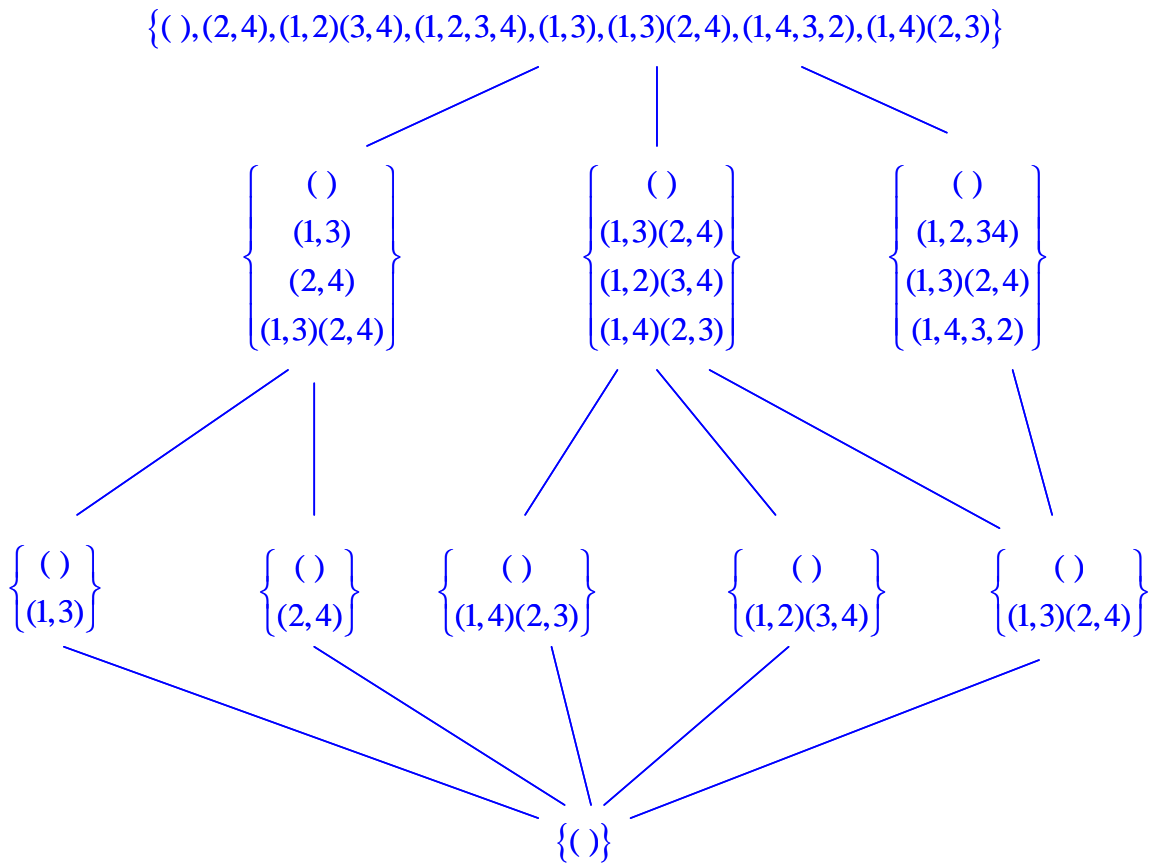


# Lesson 19

## SEMIDIRECT PRODUCTS – PRACTICE

Below is the subgroup lattice for  $D_4$ , the dihedral group of order 8 that is associated with the symmetries of a square.

$$D_4 = \{ (), (2,4), (1,2)(3,4), (1,2,3,4), (1,3), (1,3)(2,4), (1,4,3,2), (1,4)(2,3) \}$$



Let  $C_2 = \{(), (1,3)\}$  and let  $C_4 = \{(), (1,2,3,4), (1,3)(2,4), (1,4,3,2)\}$ .

1. Verify that  $C_2 \cap C_4 = \{()\}$ , that the identity is the only element in the intersection of the two subgroups.
2. Verify that  $C_2 \cdot C_4 = D_4$ , that the product of the two subgroups gives us back the entire group.

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3. Verify that  $C_2 = \{(), (1,3)\}$  is not a normal subgroup of  $D_4$ .
4. Verify that  $C_4 = \{(), (1,2,3,4), (1,3)(2,4), (1,4,3,2)\}$  is a normal subgroup of  $D_4$ .
5. Conclude that  $D_4$  is isomorphic to the semidirect product of  $C_4$  by  $C_2$ ,  $D_4 \cong C_4 \rtimes C_2$ .