

QUOTIENT STRUCTURES AND ENLIGHTENMENT

When I was in elementary school, I began reading books about yoga and meditation and spiritual masters who had attained enlightenment, and that was the beginning of a lifelong quest to achieve the same. But what is enlightenment? What do we mean by that term? There has always seemed to be an assumption by people that enlightenment is associated with terms like ultimate bliss and overcoming the subject-object dichotomy, but these terms also need clarification. Also, both these terms seem to be more descriptive of the non-verbal, right-brain than they are of the language dominated left-brain. However, it seems that “enlightenment” should also have a left-brain component because, otherwise, if there is no change in the left-brain, then one might return from these ecstatic experiences with all the same prejudices and misconceptions as before. And, indeed, there are many spiritual giants who say that enlightenment depends entirely on how our left-brain interprets things, and one could argue that this is the whole thrust of *jnana yoga*, the branch of yoga that attempts to achieve enlightenment entirely through logic and reasoning. Also when I was young, gurus and other spiritual practitioners seemed to place great emphasis on the experience of happiness and bliss, but in the process this makes pain and unhappiness the enemy, and it is difficult to attain oneness when one is surrounded by enemies. Thus, in my eyes, the pursuit of greater happiness is not always the same as the pursuit of enlightenment, but, nonetheless, the two still seem intertwined in some way, and, hence, there is still much to reconcile and to explore.

The prayer of the religious mystic is often something like, “Make me one with everything!” This prayer, however, raises two very important questions. Namely, what do we mean by everything, and what do we mean by oneness? We can address both of those questions by appealing to mathematics. In mathematics we usually use the term *set* as a synonym for *collection*, and this, for example, is the intent when we talk about something like a set of china, i.e. a collection of china, and when we speak of the union of two sets, what is meant is simply that we combine one collection with another. Thus, we could characterize every object, every perception, as a set consisting of just that one object, and then *everything* could be understood as the union of all such sets or objects. Furthermore, if each set can also be thought of as an object of perception, then we can conceive of everything as being synonymous with the union of every possible set, and we might even call it the *set of all sets*. However, when mathematicians did this at the divide between the 19th and 20th centuries, they quickly ran into problems. For example, given any set we have a well-defined way of constructing another set that is larger, and this holds true for even infinite sets, and the consequence of this is that in mathematics we actually have an infinite number of infinities of different sizes. If we can find a rule that establishes a one-to-one correspondence between the members of two sets, then we say that they are of the same size, but on the other hand, if we can show that such a correspondence does not exist, then we conclude that one of the sets is larger than the others. For example, the natural numbers, $\mathbb{N} = \{1, 2, 3, \dots\}$, represents an infinity that has the same size as the set of even natural numbers, $2\mathbb{N} = \{2, 4, 6, \dots\}$, since the rule $n \mapsto 2n$ establishes the sought after one-to-one correspondence. On the other hand, we can mathematically prove that the natural numbers cannot be put into one-to-one correspondence with the set of real numbers \mathbb{R} (the infinite continuum of numbers defined by the number line), and, therefore, the two sets represent infinities of different sizes. However, since every natural number is also contained in the set of real numbers, it follows that the real numbers correspond to the larger infinity. In mathematical symbolism we can write this as $|\mathbb{R}| > |\mathbb{N}|$. Thus, in mathematics there exist infinities of different sizes. But let’s get back now to the problem

associated with the *set of all sets*. The problem is that there are ways in which it is paradoxical and self-contradictory, and perhaps the paradox that is easiest to understand is the one known as *Cantor's paradox*. In a nutshell, this paradox boils down to the fact that if the set of all sets is really a set, then nothing should be larger than it since it contains everything including all other sets. But on the other hand, as explained above, we have a well-defined procedure for constructing a larger set from any given set. But how could any set possibly be larger than everything? That is the problem and the paradox, and at the very least it tells us that the set of all sets is not a set after all since it doesn't follow the same rules that sets follow. Hence, in mathematics, we refer to this union of all possible sets as a *class* in order to emphasize that it is different in some ways from whatever a *set* is. If we now take set theory and use it as a model for perception, then in this model every possible perception will be thought of as a set and vice-versa. However, if this is the case, then the union of all things will fail to be a perception since it is paradoxical. Hence, mathematics suggests that the union or totality of all things must be intrinsically unknowable and never an object of perception in the same way that a chair or even an idea are objects of perception. The totality of all things certainly contains all things, but at the same time it is fundamentally different in ways that are beyond perception. Interestingly, this is a truth that was intuited centuries before mathematics clarified it. For example, we have the concept of the *Great Mystery* that some Native American tribes have used to refer to divinity, and in the *Tao te Ching* we read, "*The Tao that can be named is not the Tao.*" Likewise, almost two thousand years ago the Talmudic sage Rav Huna stated, "*Whatever things you see are but parts of the ways of the Holy One, blessed be He,*" the implication being that *God*, as the totality of all things, is a whole that is unknowable even though we can perceive the parts that are contained within that whole. Thus, from a mathematical perspective, we can conclude that what we think of as *God* is something that is intrinsically unknowable, and thus, to paraphrase the *Tao te Ching*, "*The God that can be named is not THE God.*"

There is another way in which we can think of the concept of union, however, both within and without the discipline of mathematics, and this other concept is implied by *Genesis 2:24*, "*Therefore shall a man leave his father and his mother, and shall cleave to his wife; and they shall be one flesh.*" Similarly, in *Matthew 19:5-6* we read, "*Therefore a man shall leave his father and his mother and hold fast to his wife, and the two shall become one flesh, so they are no longer two but one flesh.*" In mathematics if two become one, then that means that the difference between the two objects has been divided out. In other words, we are talking about a quotient structure. Hence, instead of *union* meaning to combine one collection with another in order to get a larger collection, its usual meaning within set theory, the *two becomes one* form of union always refers to forming a quotient structure. But given this notion of union, then how are we to interpret the *union of all things*? Simple! The union of all things, as a quotient that removes the differences between things, is a state in which all differences between all things have been divided out. Hence, all distinctions between all things have been removed as a result of this union. Additionally, for clarification we will use U_S to denote the usual union of sets, and U_Q to denote union as a quotient structure. Thus, we might say that $\{tree_1\} U_S \{tree_2\} = \{tree_1, tree_2\}$, but $tree_1 U_Q tree_2 = forest$, and we'll call the former the *set union* and the latter the *quotient union*.

As a model for awareness, the quotient union U_Q creates a quotient structure in which the difference between two or more perceptions has been divided out, and in terms of perception, the quotient union of all things results in a state in which all distinctions have been removed including the subject-object dichotomy, what many would characterize as *pure awareness* or *consciousness-without-an-object*. If we explore this further, then we might conclude that awareness acts like the number 0 in addition or the null

set \emptyset under traditional set union. In other words, just as $0 + 1 = 1$ and $\emptyset \cup_S A = A$, so does *awareness* \cup_Q *object* = *object*. Hence, every perception can be thought of as a *quotient union* between awareness and the object, and the *quotient union* of all things is a state of *pure awareness* or *consciousness-without-an-object* since the distinctions between all objects have been removed. In symbols we could write this as *awareness* \cup_Q (*all objects*) = *awareness* \cup_Q *awareness* = *awareness*. Furthermore, there is a definite aspect of nullity to the quotient union of all objects, \cup_Q (*all objects*), since in this state no distinctions exist between anything, thus making it the ultimate unknowable. However, nullity is also implied by the union of all sets, \cup_S (*all sets*), since the paradoxes involved with this concept imply that it is also unknowable as any sort of collection. Hence, we might also represent these conclusions by the equations *awareness* = \emptyset and $\emptyset = U$. In other words, in the province of the mind there is no distinction between awareness, the Void, and the union of all things (both \cup_S and \cup_Q).

Within the Void there is no separation between things, and, hence, it represents the ultimate oneness or non-duality. Likewise, it is natural to equate *love* with *oneness* as it implies the phrase *two become one* in the Biblical verses quoted above. Hence, there is also a sense that as we perform *quotient unions* of more and more things, we also experience more and more love and oneness. It is for this reason that states of consciousness that exhibit more love and oneness are frequently characterized as *spiritual* while states at the opposite end of the spectrum are not. However, this incremental approach to *oneness* can also be a trap. In other words, by making a distinction between so-called spiritual states and non-spiritual states we are also setting up one to be *good* and the other to be *bad*, and this results in a less-than-enlightened quest to exalt one and suppress the other. Consequently, eating from the *Tree of good and evil* brings about not enlightenment, but instead an ongoing battle between good and evil that leads to an inevitable fall into duality. However, consider also *Isaiah 45:7*, “*I form the light, and create darkness; I make peace, and create evil; I the Lord do all these things.*” If we can understand that *light* and *dark* come from the same source, then we have a chance to escape duality by focusing on that source.

We are now in a position to provide at least somewhat of an answer to the questions that we posed at the top as to what enlightenment is. In particular, is it a right-brain experience, or is it a left-brain understanding? In my estimation, it has to be both! In other words, so-called *enlightenment* should be both a state of absolute oneness that is experienced and a state that is properly understood by the intellect. Otherwise, it is incomplete, and that is where the majority seem to be. In other words, the majority seem to be caught up in the noble struggle of good versus evil, but that very struggle itself simply perpetuates the conflict. To me, a path that leads away from this duality is found by considering more deeply the equation *awareness* \cup_Q *object* = *object*. The interpretation of this equation is simply that when we turn our awareness to some object, we perceive that object. However, since the *quotient union* removes the distinction between the objects being united, it makes just as much sense to write *awareness* \cup_Q *object* = *awareness*. Written this way our equation is a statement that every act of perception is also a uniting of awareness with an object, and each object perceived both points back to and disappears into awareness at the moment of perception. Thus, when fully understood, nothing is an obstacle to *enlightenment* or the *awareness of awareness* since every perception, both good or bad, points us back to that oneness. Hence, rather than engaging in the eternal struggle of good versus evil and dwelling in a state of near-but-not-complete enlightenment, I find it better to let every perception lead me back to that *oneness*. A similar technique has been described by Ramana Maharshi who urged people to focus on finding an answer to the question, “Who am I?” and by the Zen Buddhist masters who asked

the question, “What was your face before you were born?” Both questions urge a person to redirect awareness back upon itself, and, again, this focus on self may be summed up in the equation $awareness \cup_Q awareness = awareness$. In summary, the enlightened will see both good and evil as distractions from oneness. As the author of *Ecclesiastes* says (in more literal translations), “*Vapor of vapors, all is vapor!*” All perception is, indeed, nothing but vapor or hot air, but nonetheless, it is through our perceptions that we learn about various things, and learning defines a purpose for perception. Furthermore, even if *good* and *evil* are, in the long run, both empty experiences, good is nonetheless more pleasant than evil, and its pursuit brings one, in a sense, closer to enlightenment even if it, too, is ultimately just another addiction to be overcome. Thus, all things being both equal and equally empty, there is nothing lost and something gained by always choosing good over evil.

People often look upon enlightenment in a *discrete* or *binary* fashion. In other words, you’re either enlightened or you aren’t, and enlightenment comes in a flash! However, quantum physics has taught us that everything is characterized by paradoxical opposites, and thus, we can also think of enlightenment as a process that proceeds along a *continuum* that is *non-binary*. Looked at in this fashion, we can further analyze enlightenment through our *cycle of becoming* as described in a previous chapter. Through the lens of this cycle, enlightenment is a process that unfolds through stages that begin with an initial vision and that culminates in the inevitable union. Additionally, when viewed as a process, we might say that the moment of *enlightenment* corresponds to the full moon phase of this process. In other words, the “Aha!” moment of illumination signifies the end of only the first half of our journey! Remaining is the assimilation of this epiphany and its consequences into our lives. And so who is enlightened, and am I enlightened? I believe the correct answer to these questions is everyone and no one, and yes and no. Everything is perfect and in a state of oneness if we but realize it, and as William Blake wrote, “If the doors of perception were cleansed everything would appear to man as it is, infinite.” If we can but see the divinity contained within awareness and realize that there is no part of our experience that is not permeated by that awareness, then we are probably enlightened enough. Furthermore, this is the whole purpose of both life and astrology, to create greater and greater wholes by forming more and more quotient structures via quotient unions, and by doing so, we achieve our purpose for being here, and that is when we finally partake of the *Tree of Life*.

In summary,

- *God* or the *totality of all things* is essentially unknowable in the usual sense, and for that reason it makes just as much sense to say that *God* is evil as it does to say that *God* is good, and to say that *God* does not exist is just as truthful as the statement that *God* does exist!
- While *good* can bring us, in a sense, closer to enlightenment, ultimately both *good* and *evil* can be distractions or obstacles to a final awareness of the *self*.
- Nonetheless, *good* is preferred over *evil* simply because it makes the journey easier for everyone.
- If you experience *good*, then ask yourself who experiences good? If you experience *happiness*, then ask yourself who is happy? And even if you experience *unhappiness*, then still ask yourself who is unhappy? Inquiries such as these can eventually lead you back to your true self.
- Every quotient structure, every “Aha” moment, is a creative act in which the many unite to become the one. However, most of us get caught up in living a life where we are constantly reacting to the various things around us, and this just reinforces our bondage. Instead, focus on being creative. We are here not to be constantly reacting out of fear or desire to our various

perceptions, but to learn and to grow, and each level of growth is punctuated by a moment of “Aha!” Thus, focus not on those fears and distracting desires, but instead focus on what you are here to create. That is what matters the most, and if you do this for yourself, then you will also simultaneously be helping others, too.

- The *cycle of becoming* describes the process of union, and the moment of “Aha” or enlightenment is only the halfway point of this cycle. It is only the point at which we become the *hanged man* as in the tarot card where a man hanging upside down is suddenly seeing the world from a very different perspective. However, following this change of perspective is a process of assimilation that also has to occur.
- And who is enlightened? Ultimately and paradoxically, everyone and no one. Enlightenment is not an object of perception, but rather a correct understanding of perception. And while states of happiness and oneness will invariably be experienced along the way, the correct understanding of perception transcends all states of being.
- Enlightenment is not oneness, but the correct understanding of oneness. Nonetheless, one must experience oneness before one can understand it.
- The *union of all thing* can be understood with objects present or without objects present, and it can be known for either what we call an extended period of time or just for an instant. It can be accompanied by an aura of light and wisdom, or it can come with no such external clues. If an enlightened person perceives happiness, then that happiness is to them just a revelation of the joy of enlightenment, and if, instead, they perceive evil, then to them that evil is just a passing phase that has no substance, and then they will let even that perception of evil point them back to the unity found within awareness.
- The equation $awareness = awareness \cup_Q object = object$ is just a formal way of saying something that is quite profound when experienced. Namely, that once one realizes the impossibility of separating, at the moment of perception, the perceiving awareness from the object perceived, then one begins to realize that unity and diversity are just two sides of the same inconceivable coin, and at this point it matters not which side of the coin one is focused on. Regardless of whether one is perceiving oneness or multiplicity, one still understands the underlying “coin,” the true nature of things.
- In the aftermath of enlightenment, one’s awareness will continue to occupy the sock puppet known as the body until that bundle of energy has run its course. However, perceptions, both good and bad, will then be like scenes in a movie. They will all be observed and all point back to the observer, but they won’t be so strongly identified with as they were before. If the perception is joyous, then one will revel in the joy of it, and if it is not so joyous, then one will simply observe its passing, nudge it toward joy if possible, and always let even it redirect one’s self back toward *self-awareness*.
- And am I enlightened? Well, again the truth is that I am neither more enlightened nor less enlightened than anyone else. I know what I know, and I seek to keep knowing. Even for the so-called enlightened, there is still no end to what one may learn while incarnated within this *world of separation*. And even while I perceive what is often seen as the current sad state of the world, I also know that each perception is actually just another face of the whole since nothing ever occurs separate from the wholeness of consciousness. Thus, even though I am no different from anyone else, still feel free to call me *Grumpy Buddha* if it pleases you!