WORK - ANSWERS

Determine the work, $\int_C \vec{F} \bullet d\vec{r}$, done by each vector field below along the indicated path.

1. $\vec{F} = x\hat{i} + y\hat{j}$ and C is the line segment from $(1,2)$ to $(5,10)$.

$\vec{F} = x\hat{i} + y\hat{j},\ P = x,\ Q = y$

$$\begin{matrix} x = 1 + 4t \\ y = 2 + 8t \\ 0 \le t \le 1 \end{matrix} \Rightarrow \begin{matrix} \dfrac{dx}{dt} = 4 \\ \dfrac{dy}{dt} = 8 \end{matrix}$$

$$\int_C \vec{F} \bullet d\vec{r} = \int_C P dx + Q dy = \int_a^b \left(P\frac{dx}{dt} + Q\frac{dy}{dt} \right) dt = \int_0^1 \left[(1+4t)4 + (2+8t)8 \right] dt$$

$$= \int_0^1 \left(20 + 80t \right) dt = 20t + 40t^2 \Big|_0^1 = 60$$

2. $\vec{F} = x\hat{i} + y\hat{j}$ and C is the line segment from $(5,10)$ to $(1,2)$.

$\vec{F} = x\hat{i} + y\hat{j},\ P = x,\ Q = y$

$$\begin{matrix} x = 5 - 4t \\ y = 10 - 8t \\ 0 \le t \le 1 \end{matrix} \Rightarrow \begin{matrix} \dfrac{dx}{dt} = -4 \\ \dfrac{dy}{dt} = -8 \end{matrix}$$

$$\int_C \vec{F} \bullet d\vec{r} = \int_C P dx + Q dy = \int_a^b \left(P\frac{dx}{dt} + Q\frac{dy}{dt} \right) dt = \int_0^1 \left[(5-4t)(-4) + (10-8t)(-8) \right] dt$$

$$= \int_0^1 \left(-100 + 80t \right) dt = -100t + 40t^2 \Big|_0^1 = -60$$

3. $\vec{F} = -x\hat{i} - y\hat{j}$ and C is the line segment from $(1,2)$ to $(5,10)$.

$\vec{F} = x\hat{i} + y\hat{j},\ P = -x,\ Q = -y$

$$\begin{aligned} x &= 1+4t \qquad \frac{dx}{dt} = 4 \\ y &= 2+8t \Rightarrow \\ 0 &\le t \le 1 \qquad \frac{dy}{dt} = 8 \end{aligned}$$

$$\int_C \vec{F} \bullet d\vec{r} = \int_C P dx + Q dy = \int_a^b \left(P\frac{dx}{dt} + Q\frac{dy}{dt} \right) dt = \int_0^1 \left[(-1-4t)4 + (-2-8t)8 \right] dt$$

$$= \int_0^1 \left(-20 - 80t \right) dt = -20t - 40t^2 \Big|_0^1 = -60$$

4. $\vec{F} = x\hat{i} + y\hat{j}$ and C is the unit circle oriented counterclockwise.

$\vec{F} = x\hat{i} + y\hat{j},\ P = x,\ Q = y$

$$\begin{aligned} x &= \cos t \qquad \frac{dx}{dt} = -\sin t \\ y &= \sin t \Rightarrow \\ 0 &\le t \le 2\pi \qquad \frac{dy}{dt} = \cos t \end{aligned}$$

$$\int_C \vec{F} \bullet d\vec{r} = \int_C P dx + Q dy = \int_a^b \left(P\frac{dx}{dt} + Q\frac{dy}{dt} \right) dt = \int_0^{2\pi} \left[\cos t(-\sin t) + \sin t \cos t \right] dt$$

$$= \int_0^{2\pi} 0\, dt = 0$$

5. $\vec{F} = -y\hat{i} + x\hat{j}$ and C is the unit circle oriented counterclockwise.

$$\vec{F} = -y\hat{i} + x\hat{j},\ P = -y,\ Q = x$$

$$\begin{array}{l} x = \cos t \\ y = \sin t \Rightarrow \\ 0 \le t \le 2\pi \end{array} \quad \begin{array}{l} \dfrac{dx}{dt} = -\sin t \\ \dfrac{dy}{dt} = \cos t \end{array}$$

$$\int_C \vec{F} \bullet d\vec{r} = \int_C Pdx + Qdy = \int_a^b \left(P\frac{dx}{dt} + Q\frac{dy}{dt} \right) dt = \int_0^{2\pi} \left[(-\sin t)(-\sin t) + \cos t \cos t \right] dt$$

$$= \int_0^{2\pi} 1 dt = 2\pi$$

6. $\vec{F} = -y\hat{i} + x\hat{j}$ and C is the unit circle oriented clockwise.

$$\vec{F} = -y\hat{i} + x\hat{j},\ P = -y,\ Q = x$$

$$\begin{array}{l} x = \cos t \\ y = -\sin t \Rightarrow \\ 0 \le t \le 2\pi \end{array} \quad \begin{array}{l} \dfrac{dx}{dt} = -\sin t \\ \dfrac{dy}{dt} = -\cos t \end{array}$$

$$\int_C \vec{F} \bullet d\vec{r} = \int_C Pdx + Qdy = \int_a^b \left(P\frac{dx}{dt} + Q\frac{dy}{dt} \right) dt = \int_0^{2\pi} \left[(\sin t)(-\sin t) + \cos t(-\cos t \_ \right] dt$$

$$= \int_0^{2\pi} \left(-\sin^2 t - \cos^2 t \right) dt = -\int_0^{2\pi} dt = -2\pi$$

7. $\vec{F} = y\hat{i} - x\hat{j}$ and C is the unit circle oriented counterclockwise.

$\vec{F} = y\hat{i} - x\hat{j},\ P = y,\ Q = -x$

$$\begin{array}{l} x = \cos t \\ y = \sin t \Rightarrow \\ 0 \le t \le 2\pi \end{array} \quad \begin{array}{l} \dfrac{dx}{dt} = -\sin t \\ \dfrac{dy}{dt} = \cos t \end{array}$$

$$\int_C \vec{F} \bullet d\vec{r} = \int_C Pdx + Qdy = \int_a^b \left(P\frac{dx}{dt} + Q\frac{dy}{dt} \right) dt = \int_0^{2\pi} \left[(\sin t)(-\sin t) - \cos t(\cos t\_ \right] dt$$

$$= \int_0^{2\pi} \left(-\sin^2 t - \cos^2 t \right) dt = -\int_0^{2\pi} dt = -2\pi$$