

## UNIT TANGENTS AND NORMALS - ANSWERS

For each of the following curves, find the unit tangent and the unit normal at the indicated value for  $t$ . Also, graph the curve along with the unit tangent and normal you found.

1.  $\vec{r}(t) = \cos(t)\hat{i} + \sin(t)\hat{j}$ ,  $0 \leq t \leq 2\pi$ ,  $t = \frac{\pi}{4}$

$$\vec{r}'(t) = -\sin(t)\hat{i} + \cos(t)\hat{j}$$

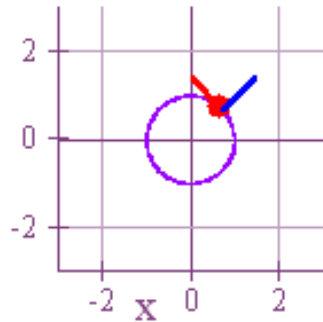
$$\|\vec{r}'(t)\| = \sqrt{(-\sin t)^2 + (\cos t)^2} = 1$$

$$T(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = -\sin(t)\hat{i} + \cos(t)\hat{j}$$

$$T\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

$$N\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

$$\vec{r}\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)\hat{i} + \sin\left(\frac{\pi}{4}\right)\hat{j} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$



$$2. \quad \vec{r}(t) = 2\cos(t)\hat{i} - 2\sin(t)\hat{j}, \quad 0 \leq t \leq 2\pi, \quad t = \frac{5\pi}{4}$$

$$\vec{r}'(t) = -2\sin(t)\hat{i} - 2\cos(t)\hat{j}$$

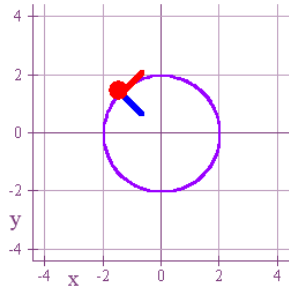
$$\|\vec{r}'(t)\| = \sqrt{(-2\sin t)^2 + (-2\cos t)^2} = 2$$

$$T(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = -\sin(t)\hat{i} + \cos(t)\hat{j}$$

$$T\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

$$N\left(\frac{5\pi}{4}\right) = \frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$$

$$\vec{r}\left(\frac{5\pi}{4}\right) = 2\cos\left(\frac{5\pi}{4}\right)\hat{i} - 2\sin\left(\frac{5\pi}{4}\right)\hat{j} = -\sqrt{2}\hat{i} + \sqrt{2}\hat{j}$$



3.  $\vec{r}(t) = (2 + 3t)\hat{i} + (1 + 4t)\hat{j}$ ,  $0 \leq t \leq 2$ ,  $t = 1$

$$\vec{r}'(t) = 3\hat{i} + 4\hat{j}$$

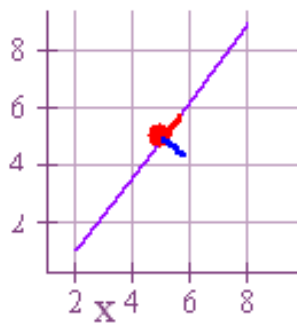
$$\|\vec{r}'(t)\| = \sqrt{3^2 + 4^2} = 5$$

$$T(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$$

$$T(1) = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$$

$$N(1) = \frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}$$

$$\vec{r}(1) = 5\hat{i} + 5\hat{j}$$



4.  $\vec{r}(t) = t\hat{i} + t^2\hat{j}$ ,  $-2 \leq t \leq 2$ ,  $t=1$

$$\vec{r}'(t) = \hat{i} + (2t)\hat{j}$$

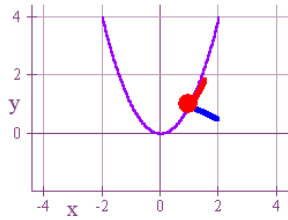
$$\|\vec{r}'(t)\| = \sqrt{1+4t^2}$$

$$T(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = \frac{1}{\sqrt{1+4t^2}}\hat{i} + \frac{2t}{\sqrt{1+4t^2}}\hat{j}$$

$$T(1) = \frac{1}{\sqrt{5}}\hat{i} + \frac{2}{\sqrt{5}}\hat{j}$$

$$N(1) = \frac{2}{\sqrt{5}}\hat{i} - \frac{1}{\sqrt{5}}\hat{j}$$

$$\vec{r}(1) = \hat{i} + \hat{j}$$



5.  $\vec{r}(t) = \sin t \hat{i} + t \hat{j}$ ,  $0 \leq t \leq 2\pi$ ,  $t = \pi$

$$\vec{r}'(t) = \cos(t)\hat{i} + \hat{j}$$

$$\|\vec{r}'(t)\| = \sqrt{1 + \cos^2 t}$$

$$\|\vec{r}'(\pi)\| = \sqrt{1 + \cos^2 \pi} = \sqrt{2}$$

$$T(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = \frac{\cos t}{\sqrt{1 + \cos^2 t}} \hat{i} + \frac{1}{\sqrt{1 + \cos^2 t}} \hat{j}$$

$$T(\pi) = -\frac{1}{\sqrt{2}} \hat{i} + \frac{1}{\sqrt{2}} \hat{j}$$

$$N(\pi) = \frac{1}{\sqrt{2}} \hat{i} + \frac{1}{\sqrt{2}} \hat{j}$$

$$\vec{r}(\pi) = \sin(\pi)\hat{i} + \pi\hat{j} = \pi\hat{j}$$

