

## TOTAL DIFFERENTIAL - ANSWERS

For each of the following functions, find the total differential.

1.  $z = f(x, y) = x^3 y^2$

$$dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = 3x^2 y^2 dx + 2x^3 y dy$$

2.  $z = f(x, y) = \sin(x^3 y^2)$

$$\begin{aligned} dz &= \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = \cos(x^3 y^2) \cdot 3x^2 y^2 dx + \cos(x^3 y^2) \cdot 2x^3 y dy \\ &= 3x^2 y^2 \cos(x^3 y^2) dx + 2x^3 y \cos(x^3 y^2) dy \end{aligned}$$

3.  $z = f(x, y) = \sqrt{x^3 y^2}$

$$\begin{aligned} dz &= \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = \frac{1}{2\sqrt{x^3 y^2}} \cdot 3x^2 y^2 dx + \frac{1}{2\sqrt{x^3 y^2}} \cdot 2x^3 y dy \\ &= \frac{3x^2 y^2}{2\sqrt{x^3 y^2}} dx + \frac{x^3 y}{\sqrt{x^3 y^2}} dy \end{aligned}$$

4.  $z = f(x, y) = \sec(x^3 y^2)$

$$\begin{aligned} dz &= \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = \sec(x^3 y^2) \tan(x^3 y^2) \cdot 3x^2 y^2 dx + \sec(x^3 y^2) \tan(x^3 y^2) \cdot 2x^3 y dy \\ &= 3x^2 y^2 \sec(x^3 y^2) \tan(x^3 y^2) dx + 2x^3 y \sec(x^3 y^2) \tan(x^3 y^2) dy \end{aligned}$$

5.  $z = f(x, y) = \tan(x^3 y^2)$

$$\begin{aligned} dz &= \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = \sec^2(x^3 y^2) \cdot 3x^2 y^2 dx + \sec^2(x^3 y^2) \cdot 2x^3 y dy \\ &= 3x^2 y^2 \sec^2(x^3 y^2) dx + 2x^3 y \sec^2(x^3 y^2) dy \end{aligned}$$