SECOND PARTIALS TEST

For each of the functions below, find the critical points and the determinant of the second partials matrix, and classify each critical point as resulting in a local maximum, local minimum, saddle point, or inconclusive. Furthermore, for each critical point (a,b), specify the coordinates (a,b,f(a,b)).

1.
$$z = f(x, y) = x^2 + y^2$$

2.
$$z = f(x, y) = x^2 - y^2$$

3.
$$z = f(x, y) = -(x^2 + y^2)$$

4.
$$z = f(x, y) = x^3 - 6x + y^3 - 9y$$

5.
$$z = f(x, y) = x^3 - 12xy - y^4$$