

## SECOND PARTIALS - ANSWERS

For each of the functions below, find  $z_{xx}$ ,  $z_{xy}$ ,  $z_{yx}$ , and  $z_{yy}$ .

1.  $z = f(x, y) = x^2 + y^2$

$$z_x = 2x$$

$$z_y = 2y$$

$$\begin{pmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

2.  $z = f(x, y) = x^2 - y^2$

$$z_x = 2x$$

$$z_y = -2y$$

$$\begin{pmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$$

3.  $z = f(x, y) = \sqrt{xy}$

$$z_x = \frac{1}{2}y(xy)^{-1/2}$$

$$z_y = \frac{1}{2}x(xy)^{-1/2}$$

$$\begin{pmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{pmatrix} = \begin{pmatrix} -\frac{1}{4}y^2(xy)^{-3/2} & \frac{1}{4}xy(xy)^{-3/2} \\ \frac{1}{4}xy(xy)^{-3/2} & -\frac{1}{4}x^2(xy)^{-3/2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{4}y^2(xy)^{-3/2} & \frac{1}{4}(xy)^{-1/2} \\ \frac{1}{4}(xy)^{-1/2} & -\frac{1}{4}x^2(xy)^{-3/2} \end{pmatrix}$$

4.  $z = f(x, y) = \ln(xy)$

$$z_x = \frac{1}{xy} \cdot y = \frac{1}{x}$$

$$z_y = \frac{1}{xy} \cdot x = \frac{1}{y}$$

$$\begin{pmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{pmatrix} = \begin{pmatrix} -\frac{1}{x^2} & 0 \\ 0 & -\frac{1}{y^2} \end{pmatrix}$$

5.  $z = f(x, y) = x^3 - 6x + y^3 - 9y$

$$z_x = 3x^2 - 6$$

$$z_y = 3y^2 - 9$$

$$\begin{pmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{pmatrix} = \begin{pmatrix} 6x & 0 \\ 0 & 6y \end{pmatrix}$$