PRELIMINARIES - ANSWERS

Find the distance between the following points. Give both an exact answer in simplest form and a decimal approximation rounded to the nearest hundredth.

1. $(1,5)$ & $(4,10)$

   distance $= \sqrt{(4-1)^2 + (10-5)^2} = \sqrt{3^2 + 5^2} = \sqrt{34} \approx 5.83$

2. $(-2,3)$ & $(-5,-8)$

   distance $= \sqrt{(-5+2)^2 + (-8-3)^2} = \sqrt{(-3)^2 + (-11)^2} = \sqrt{130} \approx 11.40$

3. $(-5,7)$ & $(17,20)$

   distance $= \sqrt{(17+5)^2 + (20-7)^2} = \sqrt{22^2 + 13^2} = \sqrt{653} \approx 25.55$

Find the equation in standard form for the circle with the given center and radius.

4. $(1,5)$ & $r = 3$

   $(x-1)^2 + (y-5)^2 = 9$

5. $(-2,3)$ & $r = 1$

   $(x+2)^2 + (y-3)^2 = 1$

6. $(17,20)$ & $r = 5$

   $(x-17)^2 + (y-20)^2 = 25$

7. $(0,0)$ & $r = 1$

   $x^2 + y^2 = 1$

Complete the square to write the equation for the circle in standard form. Identify the center and radius.

8. $x^2 + y^2 + 4x + 10y + 12 = 0$

   $(x^2 + 4x + 2^2) + (y^2 + 10y + 5^2) = -12 + 4 + 25$

   $(x+2)^2 + (y+5)^2 = 17$

   center $= (-2,-5)$

   radius $= \sqrt{17}$

9. $4x^2 + 4y^2 - 16x + 32y - 24 = 0$

$x^2 + y^2 - 4x + 8y - 6 = 0$

$(x^2 - 4x + (-2)^2) + (y^2 + 8y + 4^2) = 6 + 4 + 16$

$(x-2)^2 + (y+4)^2 = 26$

center $= (2, -4)$

radius $= \sqrt{26}$

10. $x^2 + y^2 + 41x - 107y - 43 = 0$

$$\left(x^2 + 41x + \left(\frac{41}{2}\right)^2\right) + \left(y^2 - 107y + \left(-\frac{107}{2}\right)^2\right) = 43 + \frac{1681}{4} + \frac{11,449}{4}$$

$$\left(x + \frac{41}{2}\right)^2 + \left(y - \frac{107}{2}\right)^2 = \frac{6651}{2} = 3325.5$$

center $= \left(-\frac{41}{2}, \frac{107}{2}\right)$

radius $= \sqrt{3325.5}$

11. $2x^2 + 2y^2 - 15x + 97y + 13 = 0$

$$x^2 + y^2 - \frac{15}{2}x + \frac{97}{2}y + \frac{13}{2} = 0$$

$$\left(x^2 - \frac{15}{2}x + \left(-\frac{15}{4}\right)^2\right) + \left(y^2 + \frac{97}{2}y + \left(\frac{97}{4}\right)^2\right) = -\frac{13}{2} + \frac{225}{16} + \frac{9409}{16}$$

$$\left(x - \frac{15}{4}\right)^2 + \left(y + \frac{97}{4}\right)^2 = \frac{4765}{8} = 595.625$$

center $= \left(\frac{15}{4}, -\frac{97}{4}\right)$

radius $= \sqrt{595.625}$

Complete the following tables using exact values.

12.

degrees	0	30	45	60	90
cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
sine	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
tangent	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined
cotangent	undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
secant	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	undefined
cosecant	undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

13.

radians	0	π/6	π/4	π/3	π/2
cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
sine	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
tangent	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined
cotangent	undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
secant	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	undefined
cosecant	undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Use the identity $\cos^2\theta+\sin^2\theta=1$ **&** $\cos(a+b)=\cos a\cos b-\sin a\sin b$ **to help you verify the following identities.**

14. $1+\tan^2\theta=\sec^2\theta$

$$\cos^2\theta+\sin^2\theta=1\Rightarrow\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}=\frac{1}{\cos^2\theta}\Rightarrow 1+\tan^2\theta=\sec^2\theta$$

15. $\cot^2\theta+1=\csc^2\theta$

$$\cos^2\theta+\sin^2\theta=1\Rightarrow\frac{\cos^2\theta+\sin^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}\Rightarrow \cot^2\theta+1=\csc^2\theta$$

16. $\cos^2\theta=\dfrac{1+\cos 2\theta}{2}$

$$\frac{1+\cos 2\theta}{2}=\frac{1+\cos\theta\cos\theta-\sin\theta\sin\theta}{2}=\frac{1-\sin^2\theta+\cos^2\theta}{2}=\frac{2\cos^2\theta}{2}=\cos^2\theta$$

17. $\sin^2\theta=\dfrac{1-\cos 2\theta}{2}$

$$\frac{1-\cos 2\theta}{2}=\frac{1-\left[\cos\theta\cos\theta-\sin\theta\sin\theta\right]}{2}=\frac{1-\cos^2\theta+\sin^2\theta}{2}=\frac{2\sin^2\theta}{2}=\sin^2\theta$$

18. $\sin(a+b)=\cos\left[\dfrac{\pi}{2}-(a+b)\right]$

$$\cos\left[\frac{\pi}{2}-(a+b)\right]=\cos\left(\frac{\pi}{2}\right)\cos(a+b)+\sin\left(\frac{\pi}{2}\right)\sin(a+b)=0\cdot\cos(a+b)+1\cdot\sin(a+b)$$
$$=\sin(a+b)$$

19. $\sin(a+b)=\sin a\cos b+\sin b\cos a$

$$\sin(a+b)=\cos\left[\frac{\pi}{2}-(a+b)\right]=\cos\left[\left(\frac{\pi}{2}-a\right)-b\right]=\cos\left(\frac{\pi}{2}-a\right)\cos b+\sin\left(\frac{\pi}{2}-a\right)\sin b$$
$$=\sin(a)cos(b)+\cos(a)\sin(b)=\sin(a)\cos(b)+\sin(b)\cos(a)$$

Give formulas for the following.

20. Area of a circle

$A=\pi r^2$

21. Circumference of a circle

$C=\pi d=2\pi r$

22. Area of a parallelogram

$A = (base)(height)$

23. Area of a trapezoid

$$A = \frac{(b_1 + b_2)h}{2}$$

24. Volume of a sphere

$$V = \frac{4}{3}\pi r^3$$

Find equations in slope-intercept form (if possible) for the following.

25. The line of slope 3 that passes through the point (1,5).

$y = 3(x-1)+5$

$y = 3x+2$

26. The line that passes through (-2,8) and (4,-5).

$slope = \frac{-5-8}{4-(-2)} = -\frac{13}{6}$

$y = -\frac{13}{6}(x+2)+8 = -\frac{13}{6}x - \frac{13}{3} + \frac{24}{3} = -\frac{13}{6}x + \frac{11}{3}$

$y = -\frac{13}{6}x + \frac{11}{3}$

27. The line that passes through (-2,-10) and (2,-5).

$slope = \frac{-10+5}{-2-2} = \frac{-5}{-4} = \frac{5}{4}$

$y = \frac{5}{4}(x-2)-5 = \frac{5}{4}x - \frac{5}{2} - \frac{10}{2} = \frac{5}{4}x - \frac{15}{2}$

$y = \frac{5}{4}x - \frac{15}{2}$

28. The line that passes through (-2,-10) and (-2,-5).

$slope = \frac{-10+5}{-2+2} = \frac{-5}{0} = undefined$

$x = -2$

29. The line that passes through (-2,-10) and (2,-10).

$slope = \frac{-10+10}{-2-2} = \frac{0}{-4} = 0$

$y = -10$

30. The line that passes through (-2,-10) and is perpendicular to $3x+2y=10$.

$(-2,-10)$

$3x+2y=10 \Rightarrow 2y = -3x+10 \Rightarrow y = -\frac{3}{2}x+5 \Rightarrow slope = \frac{2}{3}$

$y = \frac{2}{3}(x+2)-10 = \frac{2}{3}x + \frac{4}{3} - \frac{30}{3} = \frac{2}{3}x - \frac{26}{3}$

$y = \frac{2}{3}x - \frac{26}{3}$

Find the following.

31. $\frac{d}{dx}\cos x = -\sin x$

32. $\frac{d}{dx}\sin x = \cos x$

33. $\frac{d}{dx}\sec x = \sec x \tan x$

34. $\frac{d}{dx}\csc x = -\csc x \cot x$

35. $\frac{d}{dx}\tan x = \sec^2 x$

36. $\frac{d}{dx}\cot x = -\csc^2 x$

37. $\int \cos x \, dx = \sin x + C$

38. $\int \sin x \, dx = -\cos x + C$

39. $\int \sec x \, dx = \ln\left|\sec x + \tan x\right| + C$

40. $\int \csc x \, dx = -\ln\left|\csc x + \cot x\right| + C$

41. $\int \tan x \, dx = -\ln\left|\cos x\right| + C$

42. $\int \cot x \, dx = \ln\left|\sin x\right| + C$

Perform the indicated operations.

43. $\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}\begin{pmatrix}5 & 6\\ 7 & 8\end{pmatrix}=\begin{pmatrix}5+14 & 6+16\\ 15+28 & 18+32\end{pmatrix}=\begin{pmatrix}19 & 22\\ 43 & 50\end{pmatrix}$

44. $\begin{pmatrix}1 & 2 & 3\end{pmatrix}\begin{pmatrix}4\\ 5\\ 6\end{pmatrix}=\begin{pmatrix}4+10+18\end{pmatrix}=\begin{pmatrix}32\end{pmatrix}$

45. $\begin{pmatrix}2 & 3\\ 4 & 5\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}2x+3y\\ 4x+5y\end{pmatrix}$

46. $\begin{vmatrix}3 & 2\\ 1 & 4\end{vmatrix}=3\cdot 4-1\cdot 2=10$

47. $\begin{vmatrix}2 & 4 & 6\\ 3 & 0 & 1\\ 1 & 4 & 5\end{vmatrix}=2\begin{vmatrix}0 & 1\\ 4 & 5\end{vmatrix}-4\begin{vmatrix}3 & 1\\ 1 & 5\end{vmatrix}+6\begin{vmatrix}3 & 0\\ 1 & 4\end{vmatrix}=2(0-4)-4(15-1)+6(12-0)$

$=2(-4)-4(14)+6(12)=-8-56+72=8$

48. $\begin{vmatrix}2 & 4 & 6\\ 7 & 8 & 9\\ 9 & 8 & 8\end{vmatrix}=2\begin{vmatrix}8 & 9\\ 8 & 8\end{vmatrix}-4\begin{vmatrix}7 & 9\\ 9 & 8\end{vmatrix}+6\begin{vmatrix}7 & 8\\ 9 & 8\end{vmatrix}=2(64-72)-4(56-81)+6(56-72)$

$=2(-8)-4(-25)+6(-16)=-16+100-96=-12$

49. $\begin{vmatrix}7 & 8 & 9\\ 2 & 4 & 6\\ 9 & 8 & 8\end{vmatrix}=7\begin{vmatrix}4 & 6\\ 8 & 8\end{vmatrix}-8\begin{vmatrix}2 & 6\\ 9 & 8\end{vmatrix}+9\begin{vmatrix}2 & 4\\ 9 & 8\end{vmatrix}=7(32-48)-8(16-54)+9(16-36)$

$=7(-16)-8(-38)+9(-20)=-112+304-180=12$

50. $\begin{vmatrix}2 & 4 & 6\\ 1 & 2 & 3\\ 4 & 5 & 6\end{vmatrix}=2\begin{vmatrix}2 & 3\\ 5 & 6\end{vmatrix}-4\begin{vmatrix}1 & 3\\ 4 & 6\end{vmatrix}+6\begin{vmatrix}1 & 2\\ 4 & 5\end{vmatrix}=2(12-15)-4(6-12)+6(5-8)$

$=2(-3)-4(-6)+6(-3)=-6+24-18=0$