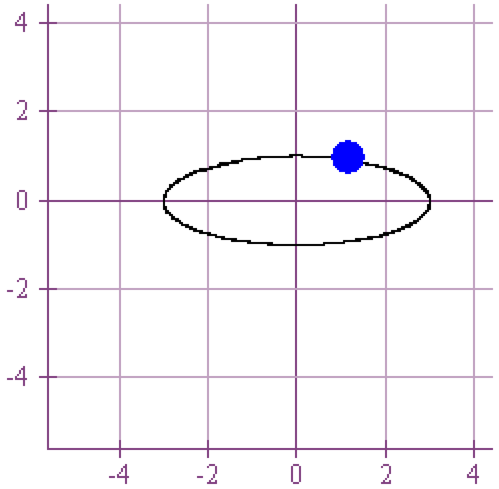
# UNIT TANGENTS AND NORMALS

Suppose we have a curve defined by a vector valued function.

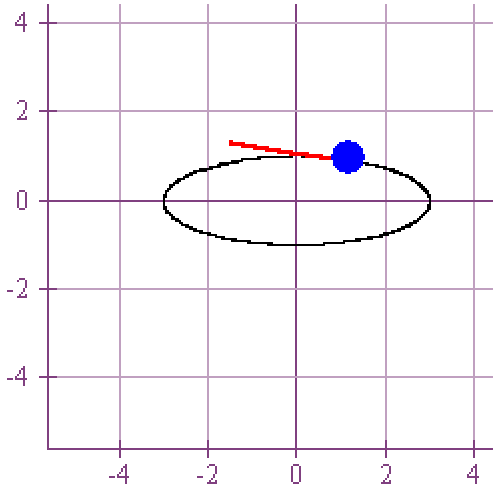
$$\vec{r}(t) = P(t)\hat{i} + Q(t)\hat{j}$$

$$a \le t \le b$$



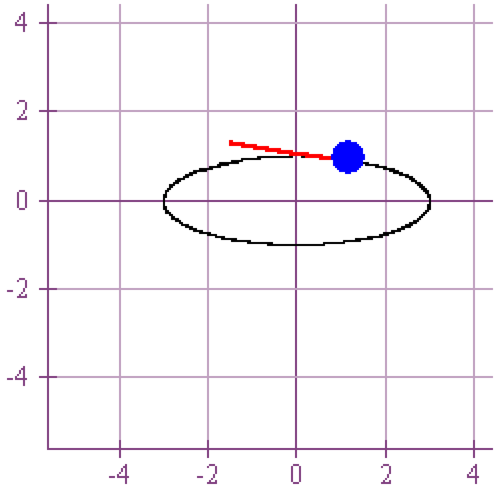
Then the derivative evaluated at a point will give us a vector tangent to the curve.

$$\vec{r}'(t) = P'(t)\hat{i} + Q'(t)\hat{j}$$



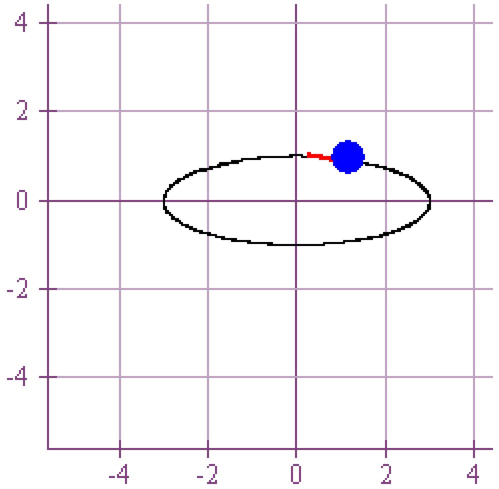
However, this vector is not necessarily a unit vector.

$$\vec{r}'(t) = P'(t)\hat{i} + Q'(t)\hat{j}$$



To get the unit tangent vector, divide *dr/dt* by its length.

$$T = \frac{\vec{r}'(t)}{\left\|\vec{r}'(t)\right\|} = \frac{d\vec{r}'/dt}{\left\|d\vec{r}'/dt\right\|}$$

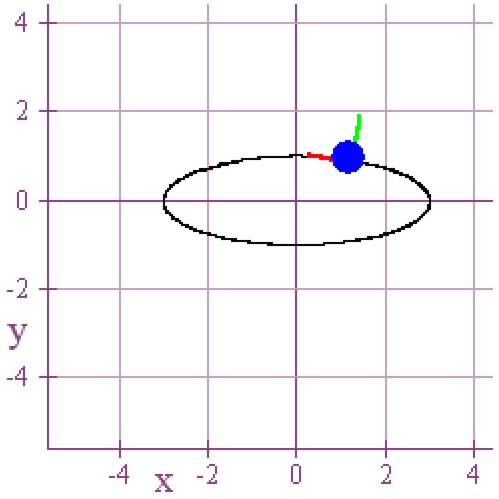


If our unit tangent vector is written in component form as,

$$T = P\hat{i} + Q\hat{j}$$

Then we define the unit normal by,

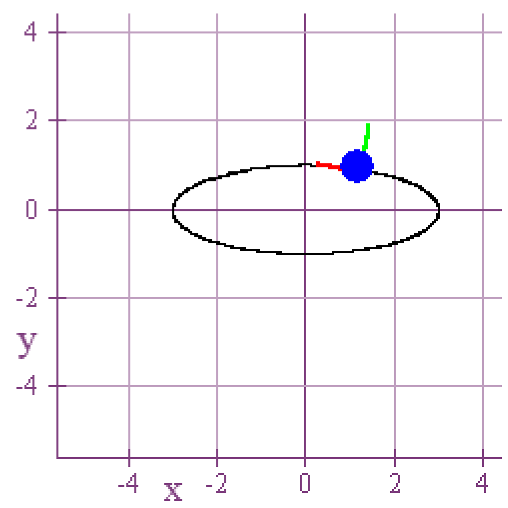
$$N = Q\hat{i} - P\hat{j}$$



This method always results in a normal vector that points to the right of the direction in which our unit tangent vector is facing.

$$T = P\hat{i} + Q\,\hat{j}$$

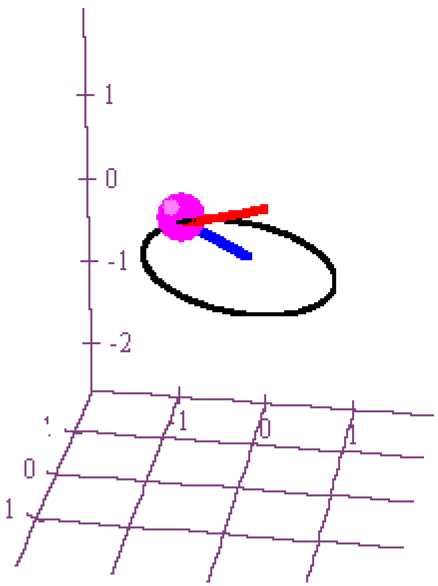
$$N = Q\hat{i} - P\,\hat{j}$$



If we have a curve in three dimensions, then things are slightly more complicated.

$$\vec{r}(t) = P(t)\hat{i} + Q(t)\hat{j} + R(t)\hat{k}$$

$$a \le t \le b$$



However, since we won't need that case for what we are going to do later, we'll skip it for now.

$$\vec{r}(t) = P(t)\hat{i} + Q(t)\hat{j} + R(t)\hat{k}$$

$$a \le t \le b$$