TAYLOR POLYNOMIAL APPROXIMATIONS

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Thus, the curvature of $f(x)$ is similar to the curvature of $P_2(x)$ at a.

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$$z = f(x,y)$$

$$P_2(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$+ \frac{f_{xx}(a,b)}{2}(x-a)^2 + f_{xy}(a,b)(x-a)(y-b) + \frac{f_{yy}(a,b)}{2}(y-b)^2$$

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This polynomial has the same value at (a,b) as the function $z = f(x,y)$, and it also has the same values at this point for the first and second partial derivatives.

Example:

$$z = f(x, y)$$

$$P_2(x, y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

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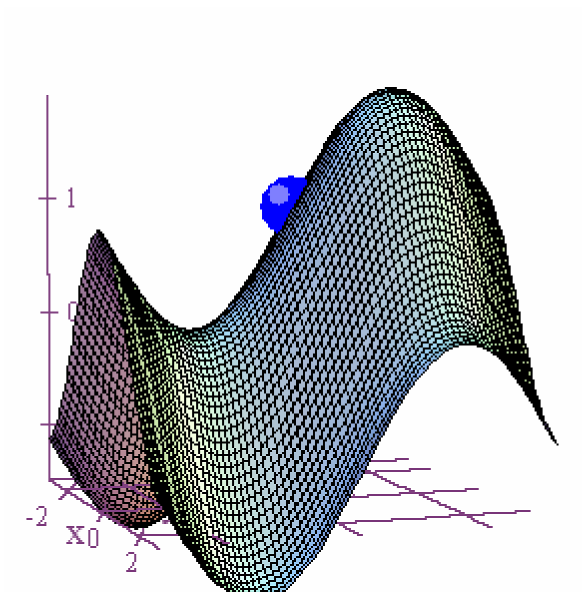
$$z = \cos(x) + \sin(y)$$

$$Q = (0,0)$$

$$P_2(x, y) = 1 + y - \frac{1}{2}x^2$$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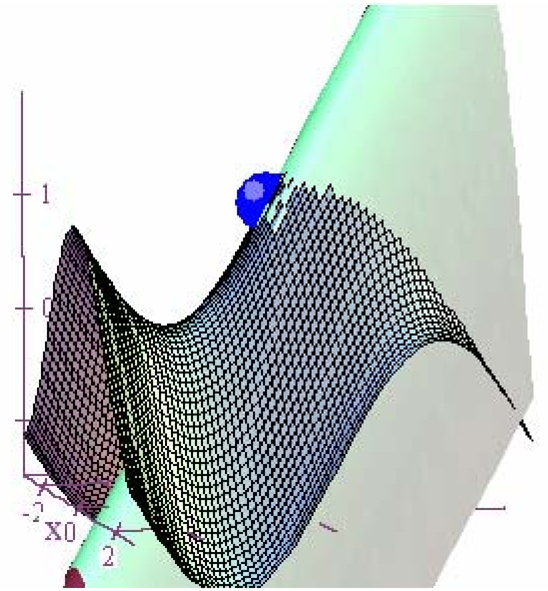
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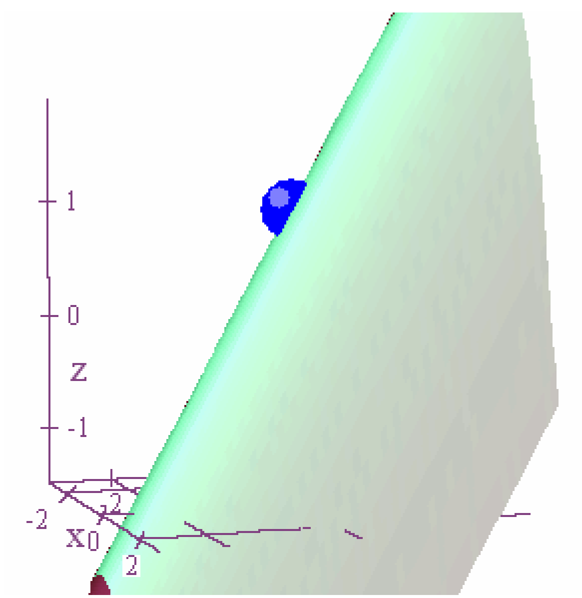
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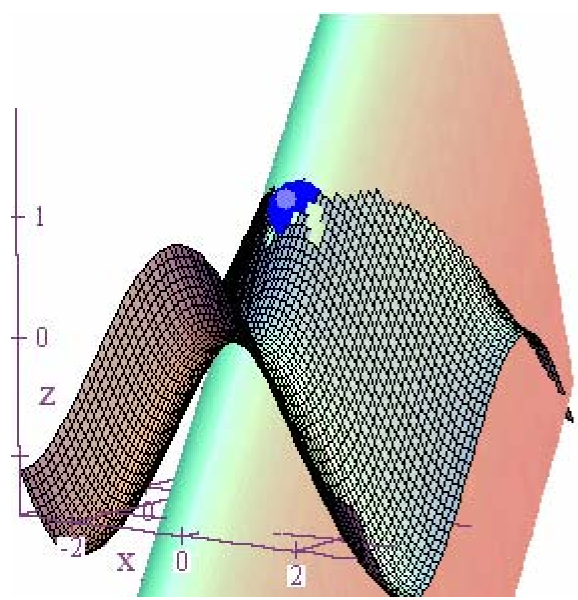
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Another way to prove the second partials test is to verify it at a critical point for the corresponding 2nd degree Taylor polynomial.