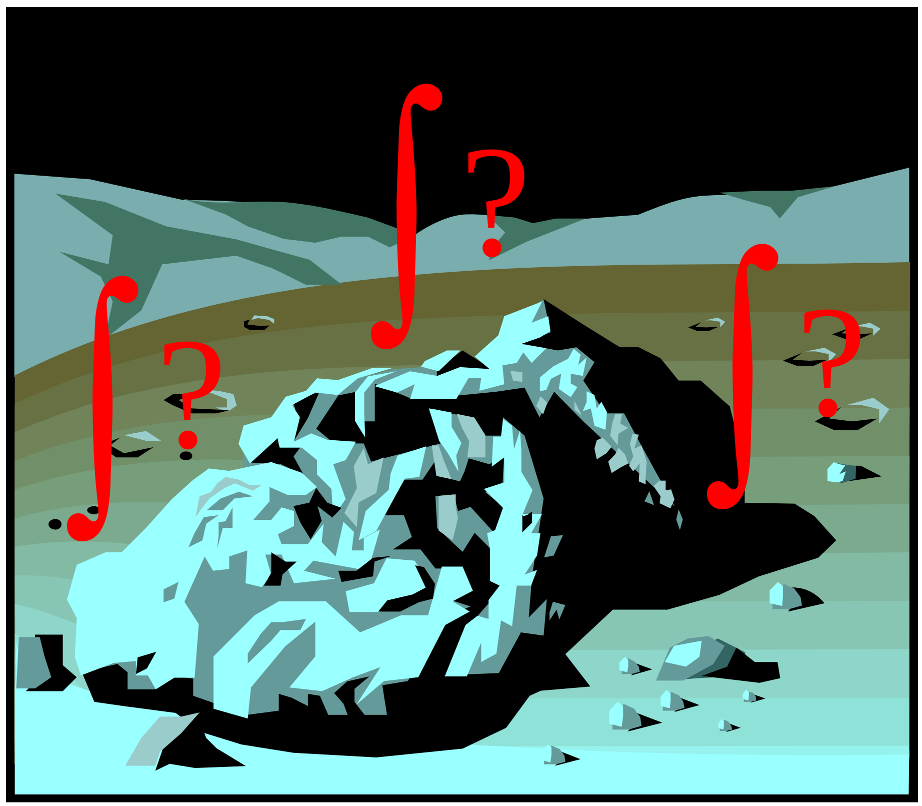
SURFACE INTEGRALS



At this point, we'll think of a surface integral as simply a function *g=g(x,y,z)* that is integrated with respect to an element of area on the surface graph of a function *z=f(x,y)* rather then an element of area in the *xy*-plane.

$$\iint_S y\, dS$$

The surface S is defined by $z = 4 - y^2$

where $0 \le x \le 12$ and $0 \le y \le 2$

If we now make the appropriate substitution for *dS*, then the surface integral becomes very easy to evaluate.

$$z = 4 - y^2$$

$$0 \le x \le 12$$

$$0 \le y \le 2$$

$$\iint_S y\,dS = \iint_R y\sqrt{z_x^2 + z_y^2 + 1}\,dA$$

$$= \int_0^{12}\int_0^2 y\sqrt{4y^2+1}\,dy\,dx$$

$$= \frac{1}{8}\int_0^{12}\int_0^2 (4y^2+1)^{1/2}\,8y\,dy\,dx = \frac{1}{8}\int_0^{12}\int_1^{17} u^{1/2}\,du\,dx \quad \begin{pmatrix} u = 4y^2+1 \\ du = 8y\,dy \end{pmatrix}$$

$$= \frac{1}{8}\int_0^{12} \frac{2u^{3/2}}{3}\Bigg|_1^{17} dx = \frac{1}{8}\cdot\frac{2\cdot 17^{3/2} - 2}{3}\int_0^{12} dx = \frac{17^{3/2} - 1}{12}\cdot 12 = 17^{3/2} - 1$$

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BEAUTIFUL!