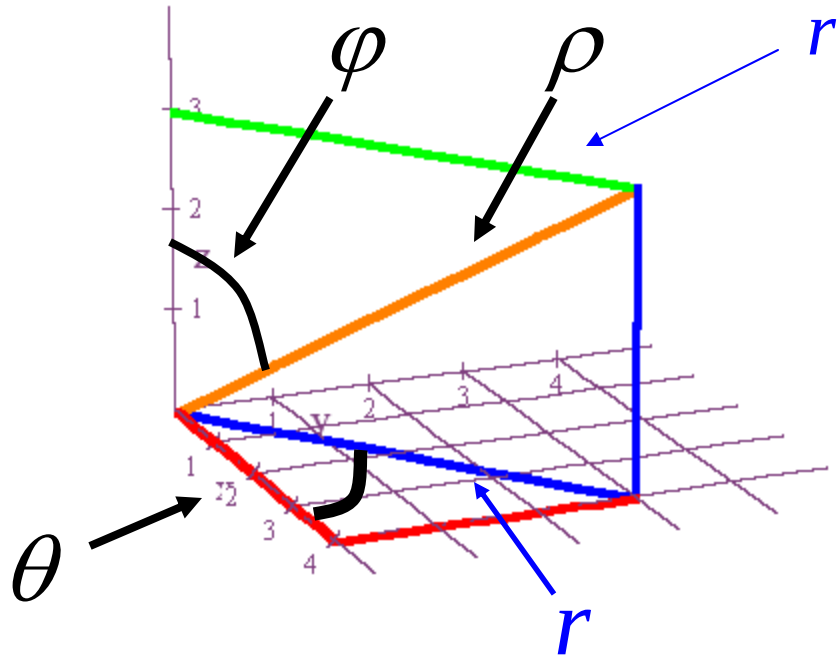
SPHERICAL COORDINATES (ρ, θ, φ)



$r = \rho \sin \varphi$

$$\rho^2 = x^2 + y^2 + z^2$$

$$\tan \theta = \frac{y}{x}$$

$$\varphi = \arccos \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$x = r\cos(\theta) = \rho \sin(\varphi)\cos(\theta)$$

$$y = r\sin(\theta) = \rho \sin(\varphi)\sin(\theta)$$

$$z = \rho \cos(\varphi)$$

$$0 \le \rho < \infty$$

$$0 \le \varphi \le \pi$$

$$0 \le \theta \le 2\pi$$

Convert from spherical coordinates to rectangular

$$(\rho,\theta,\varphi)_{\text{spherical}} = (2,\pi/2,3\pi/4)_{\text{spherical}}$$

$$(\rho\sin\varphi\cos\theta,\rho\sin\varphi\sin\theta,\rho\cos\varphi)_{\text{rectangular}} = \left(2\sin\frac{3\pi}{4}\cos\frac{\pi}{2},2\sin\frac{3\pi}{4}\sin\frac{\pi}{2},2\cos\frac{3\pi}{4}\right)_{\text{rectangular}}$$

$$= \left(0,\sqrt{2},-\sqrt{2}\right)_{\text{rectangular}}$$

Convert from rectangular coordinates to spherical

$$\left(x, y, z\right)_{\text{rectangular}} = (1, -1, -\sqrt{2})_{\text{rectangular}}$$

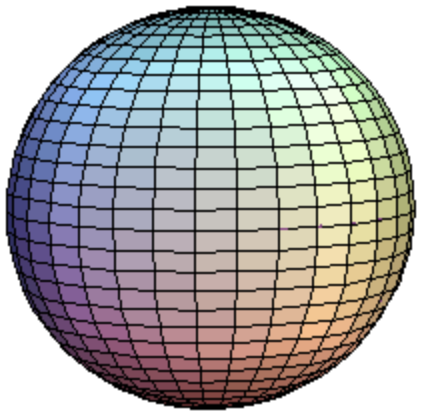
Note that with respect to *x* and *y* our point is in quadrant IV.

$$\tan\theta = \frac{y}{x} = \frac{-1}{1} \Rightarrow \theta = \tan^{-1}\left(-1\right) = -45° = -\frac{\pi}{4}$$

The related fourth quadrant angle is $\frac{7\pi}{4}$

$$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{1+1+2} = \sqrt{4} = 2 \qquad \varphi = \cos^{-1}\left(\frac{z}{\rho}\right) = \cos^{-1}\left(\frac{-\sqrt{2}}{2}\right) = \frac{3\pi}{4}$$

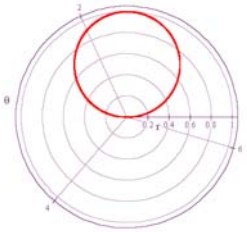
$$\left(1, -1, -\sqrt{2}\right)_{\text{rectangular}} = \left(2, \frac{7\pi}{4}, \frac{3\pi}{4}\right)_{\text{spherical}}$$



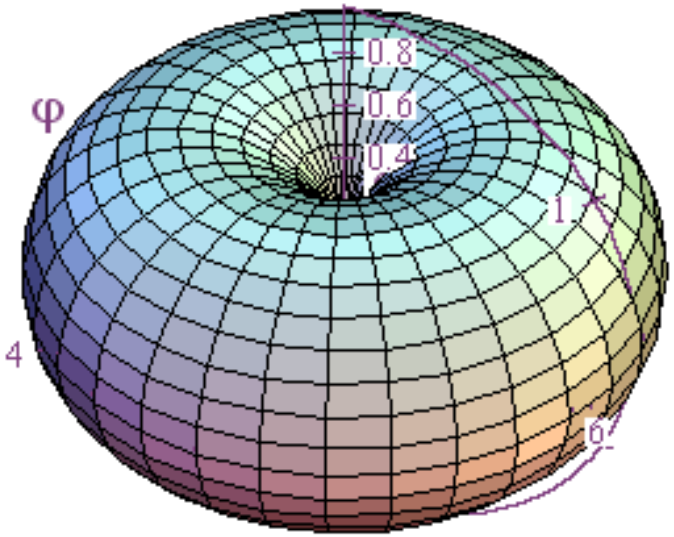
$$\rho = 5$$

$$0 \le \theta \le 2\pi$$

$$0 \le \varphi \le \pi$$



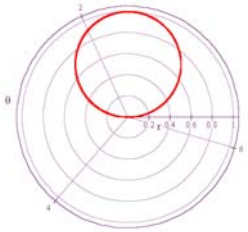
$r = \sin\theta$



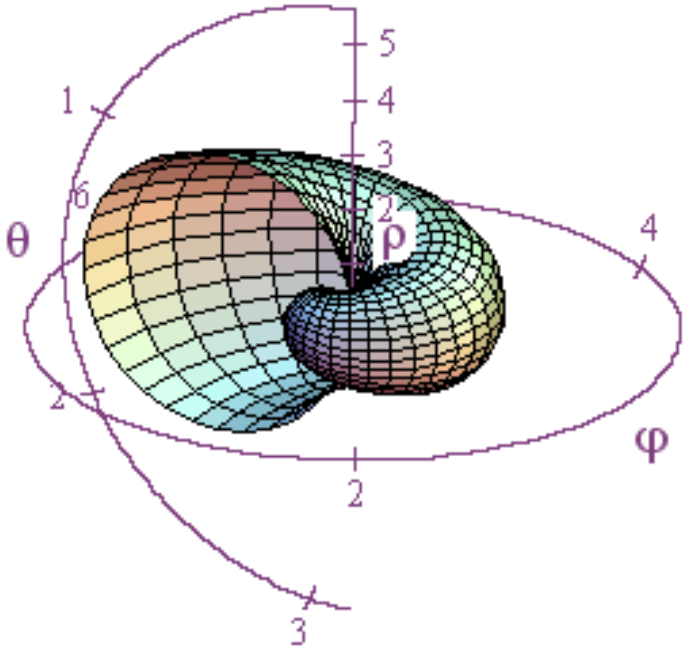
$$\rho = \sin\varphi$$

$$0 \le \theta \le 2\pi$$

$$0 \le \varphi \le \pi$$



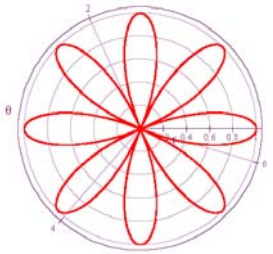
$r = \sin\theta$



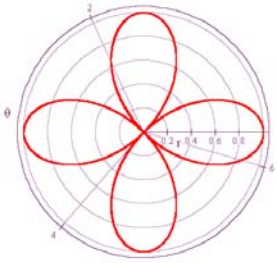
$$\rho = \sin\varphi \cdot 1.3^{\theta}$$

$$0 \le \theta \le 2\pi$$

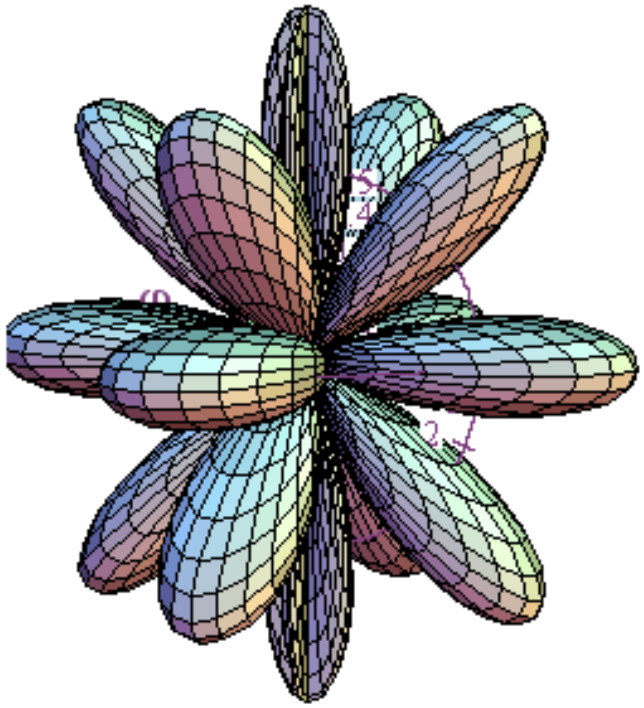
$$0 \le \varphi \le \pi$$



$r = \cos 4\theta$



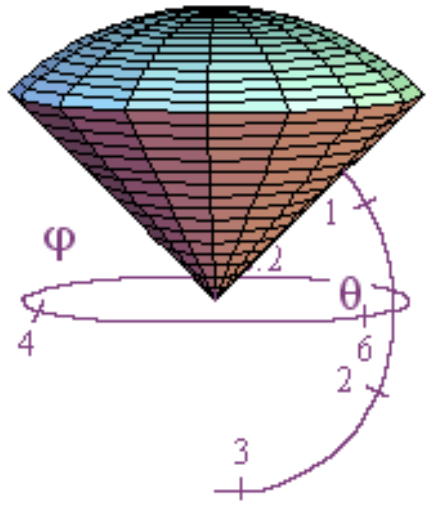
$r = \cos 2\theta$



$$\rho = \cos 4\varphi \cos 2\theta$$

$$0 \le \theta \le 2\pi$$

$$0 \le \varphi \le \pi$$



Ice Cream Top	Cone
$\rho = \sqrt{2}$	$0 \le \rho \le \sqrt{2}$
$0 \le \theta \le 2\pi$	$0 \le \theta \le 2\pi$
$0 \le \varphi \le \frac{\pi}{4}$	$\varphi = \frac{\pi}{4}$