2^{ND} ORDER PARTIAL DERIVATIVES

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Theorem: If z_{xy} and z_{yx} are continuous at (a,b), and interior point of the domain, then $z_{xy}(a,b) = z_{yx}(a,b)$.

What do the 2nd order partials tell us?

$$z = x^2 - y^2$$

$$z_x = 2x$$

$$z_y = -2y$$

$$z_{xx} = 2$$

$$z_{yy} = -2$$

In this case, they tell us that for a fixed value of y, the curve of intersection will be concave up.

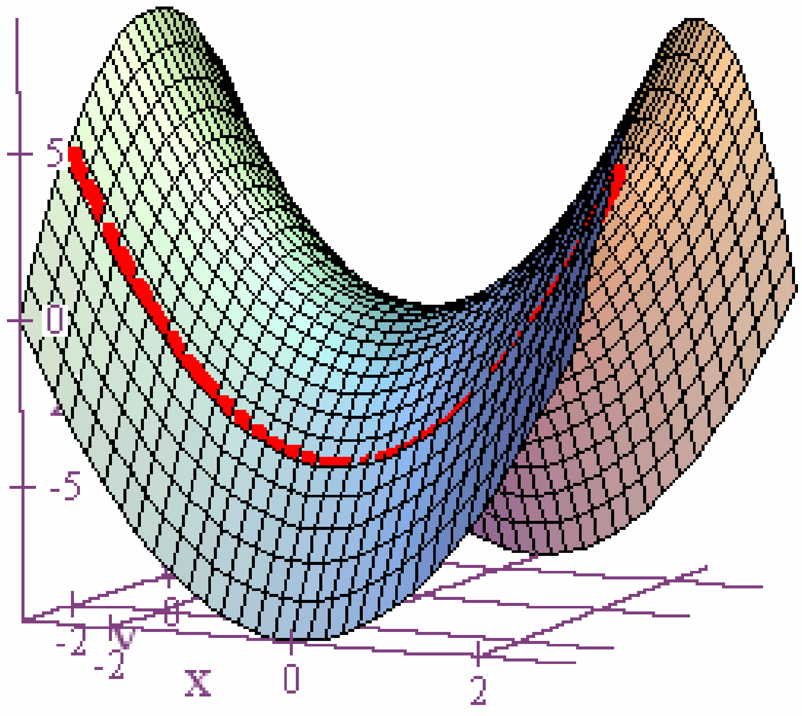
$$z = x^2 - y^2$$

$$z_x = 2x$$

$$z_y = -2y$$

$$z_{xx} = 2$$

$$z_{yy} = -2$$



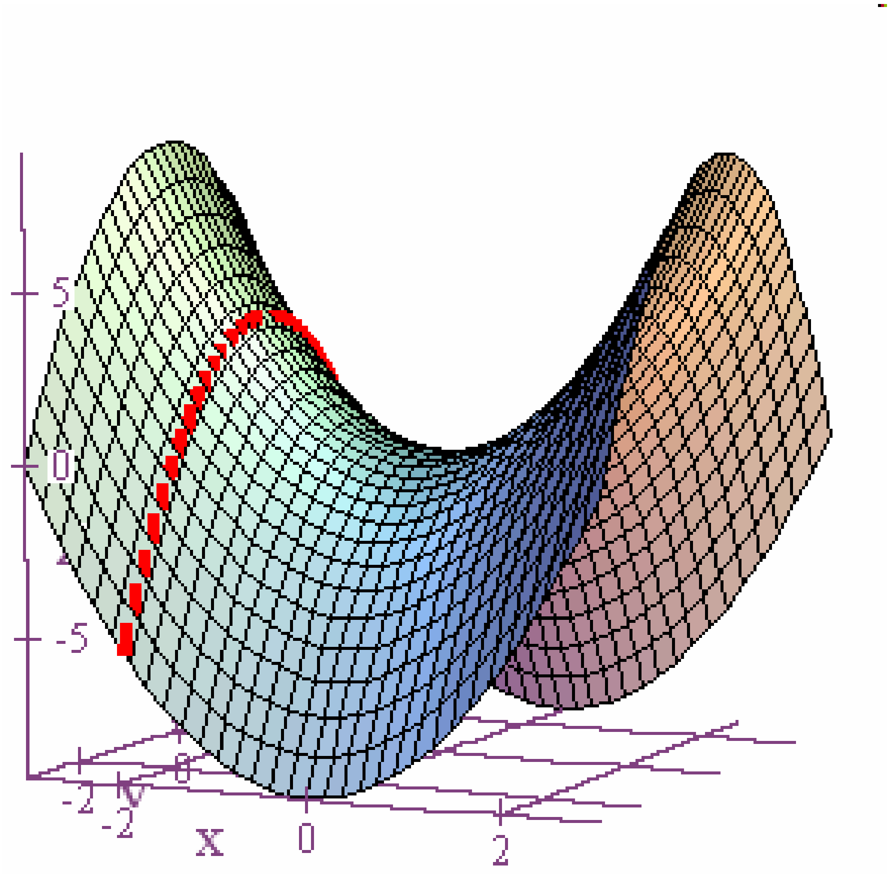
And for a fixed value of *x*,

the curve of intersection will be concave down.

$$z = x^2 - y^2$$

$$z_x = 2x \qquad z_{xx} = 2$$

$$z_y = -2y \qquad z_{yy} = -2$$



The mixed partials are trickier to visualize.

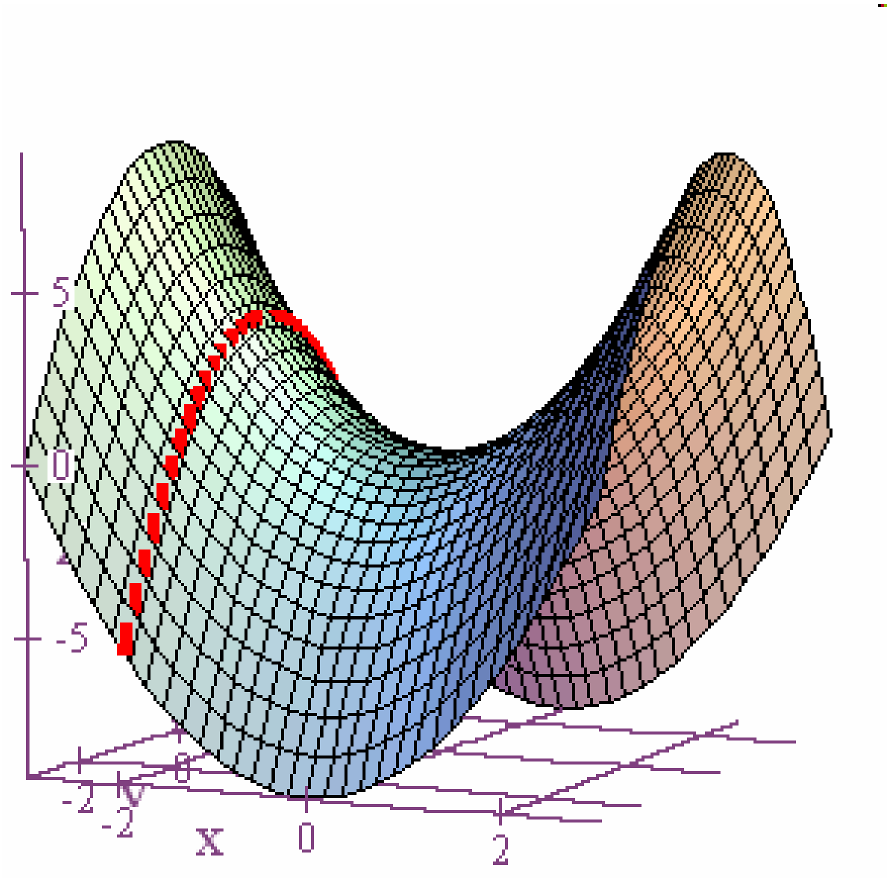
$$z = x^2 - y^2$$

$$z_x = 2x$$

$$z_y = -2y$$

$$z_{xy} = 0$$

$$z_{yx} = 0$$

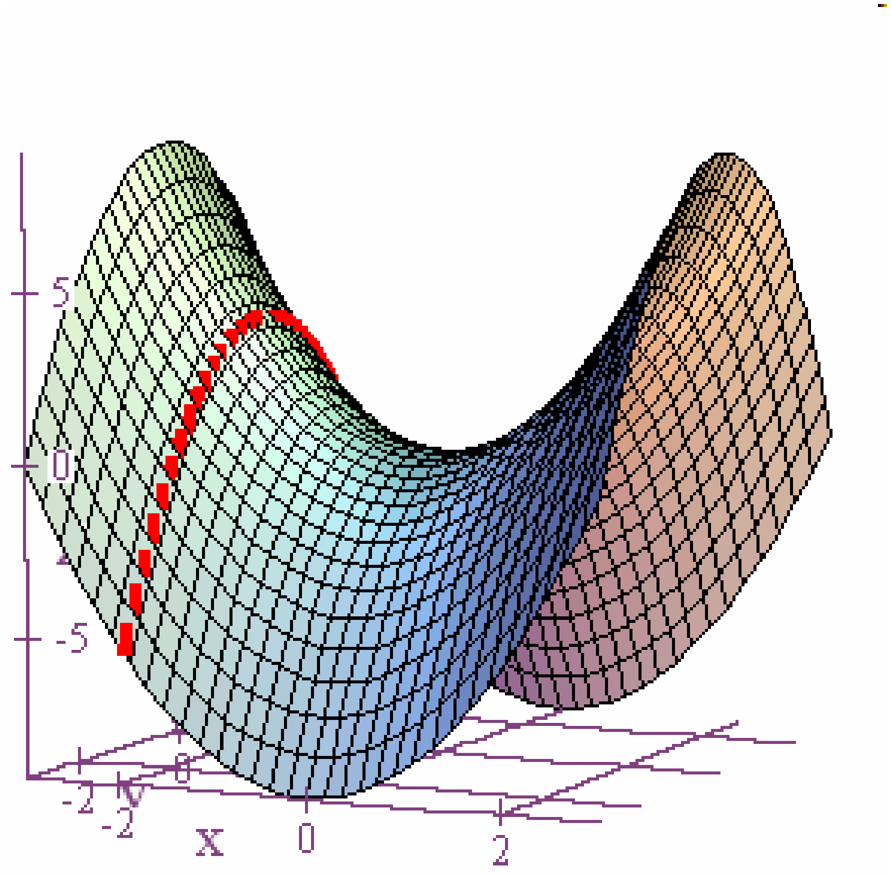


In this instance, if we find the partial first with respect to *y*, then the partial of this with respect to *x* shows how the derivative with respect to *y* changes as *x* changes.

$$z = x^2 - y^2$$

$$z_x = 2x \qquad z_{xy} = 0$$

$$z_y = -2y \qquad z_{yx} = 0$$

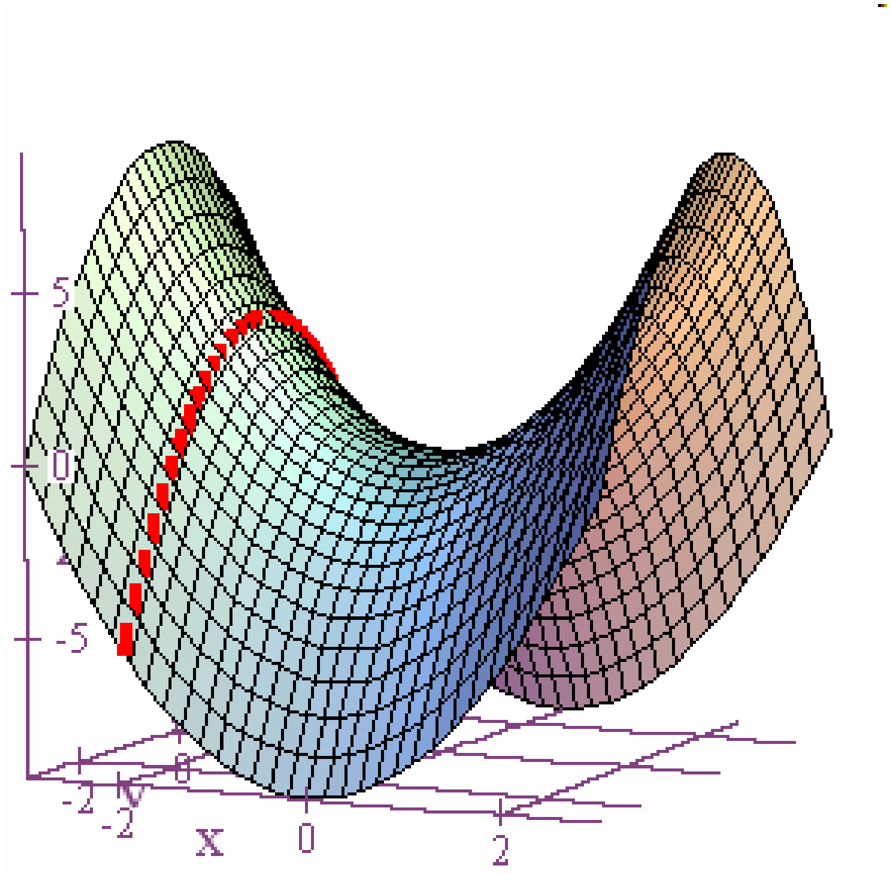


In this instance, if we find the partial first with respect to *y*, then the partial of this with respect to *x* shows how the derivative with respect to *y* changes as *x* changes. TRICKY!

$$z = x^2 - y^2$$

$$z_x = 2x \qquad z_{xy} = 0$$

$$z_y = -2y \qquad z_{yx} = 0$$



Below we see tangent lines whose slopes represent derivatives with respect to *y*.

$z = x^2 - y^2$

$z_x = 2x$

$z_y = -2y$

$z_{xy} = 0$

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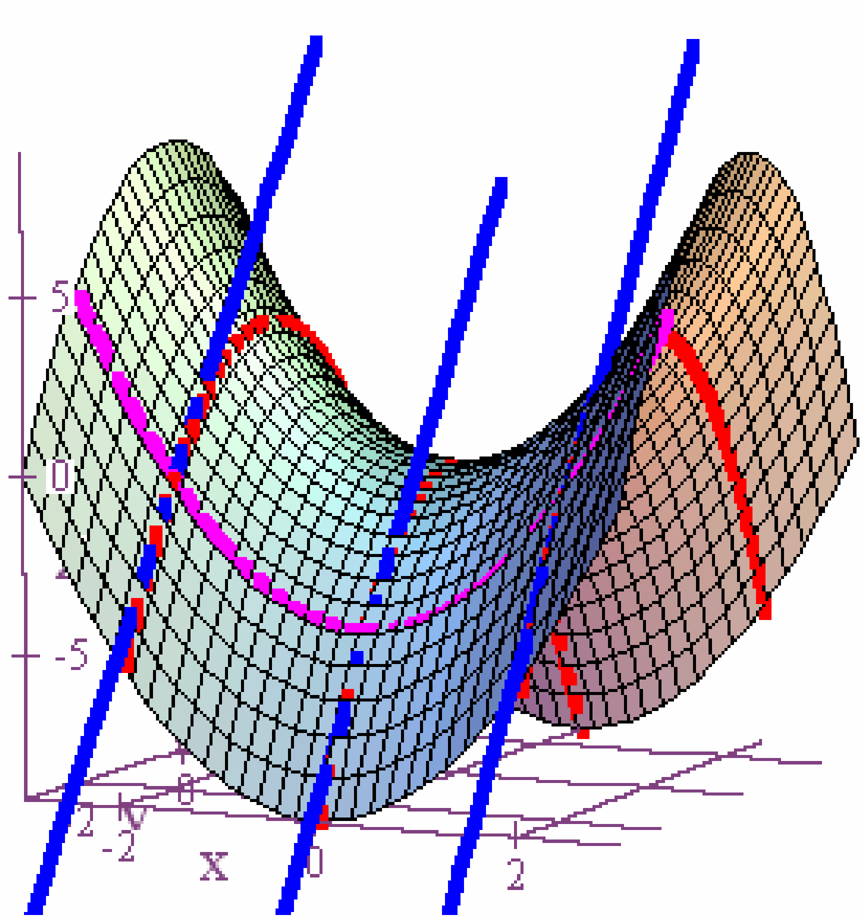

However, since $z_{yx} = 0$, these slopes do not change as x changes.

$$z = x^2 - y^2$$

$$z_x = 2x \qquad z_{xy} = 0$$

$$z_y = -2y \qquad z_{yx} = 0$$