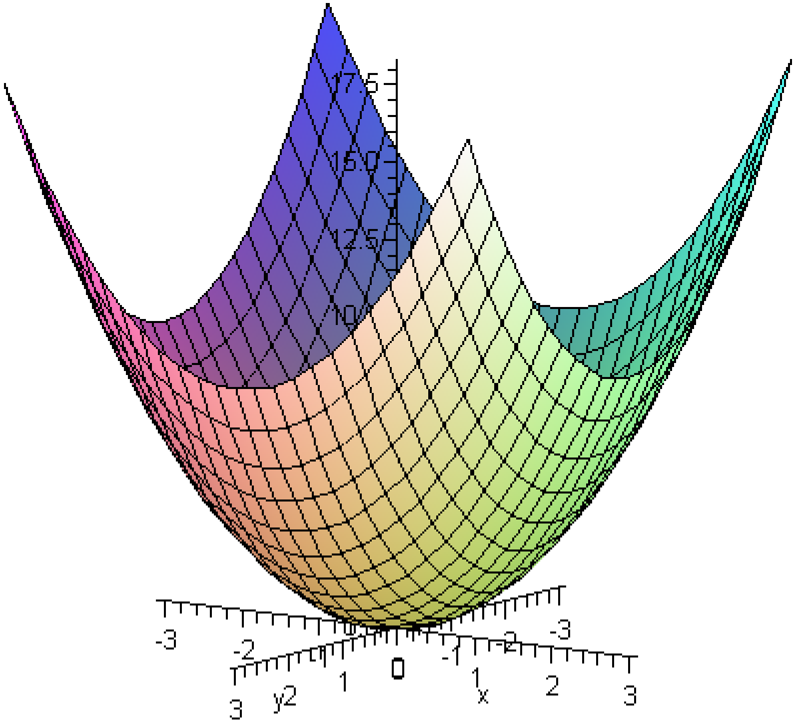
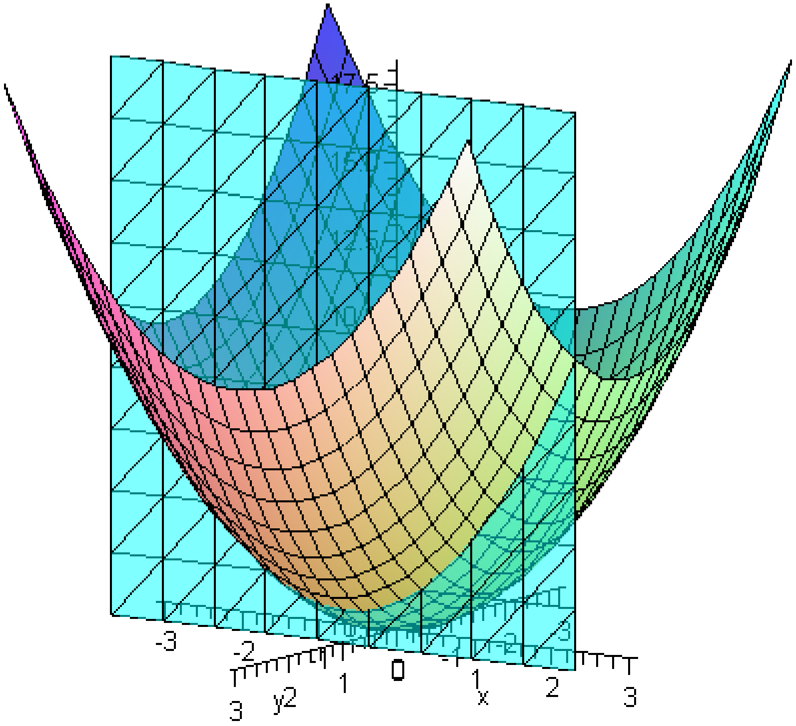
PUTTING IT ALL TOGETHER

Let's start with the graph of $z = x^2 + y^2$.

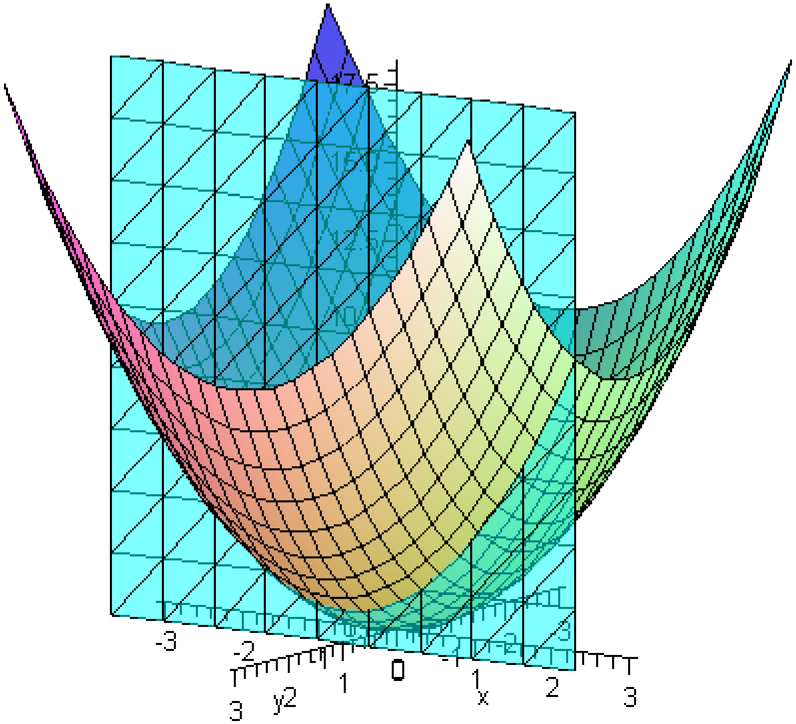


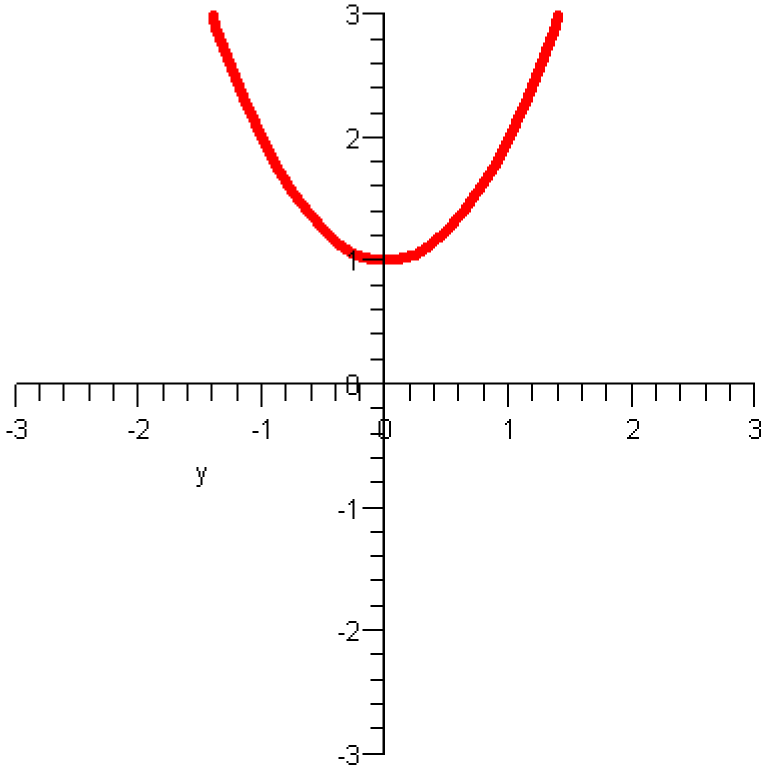
Next, let's look at the intersection of this surface with the plane $x = 1$.



$$implicitplot3d(x = 1,\ x = -3..3,\ y = -3..3,\ z = 0..18,\ color = cyan,\ transparency = 0.5,\ axes = normal);$$

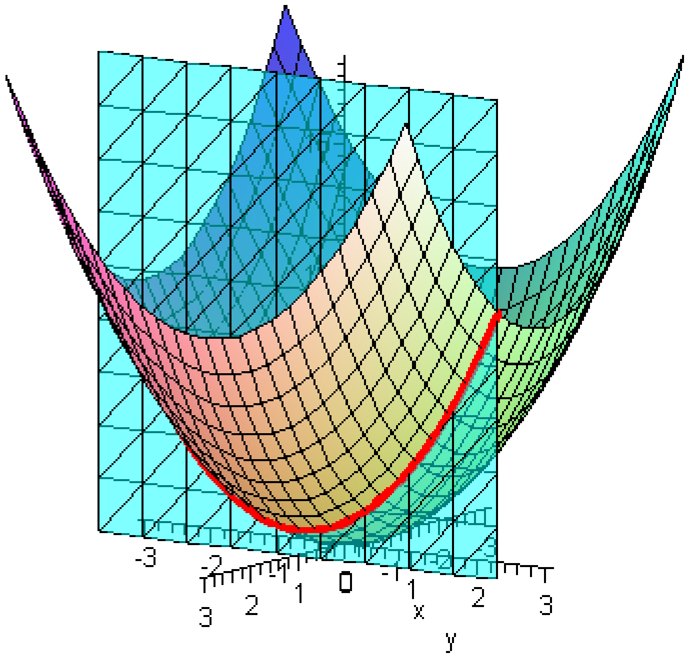
The equation in 2-dimensions for the curve of intersection is $z = 1^2 + y^2 = 1 + y^2$.



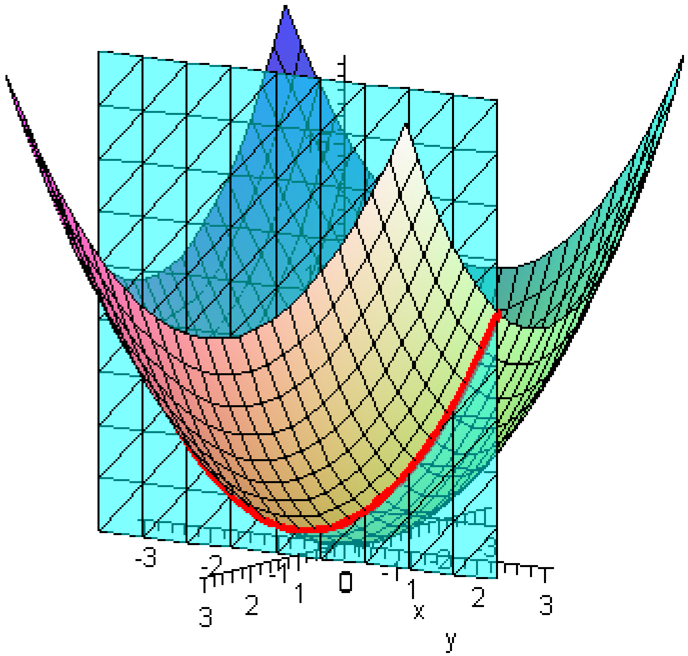


$plot(1 + y^2, y = -3..3, view = [-3..3, -3..3], thickness = 3);$

We can use parametric equations to add this curve to our graph in three dimensions.



What are the appropriate parametric equations?



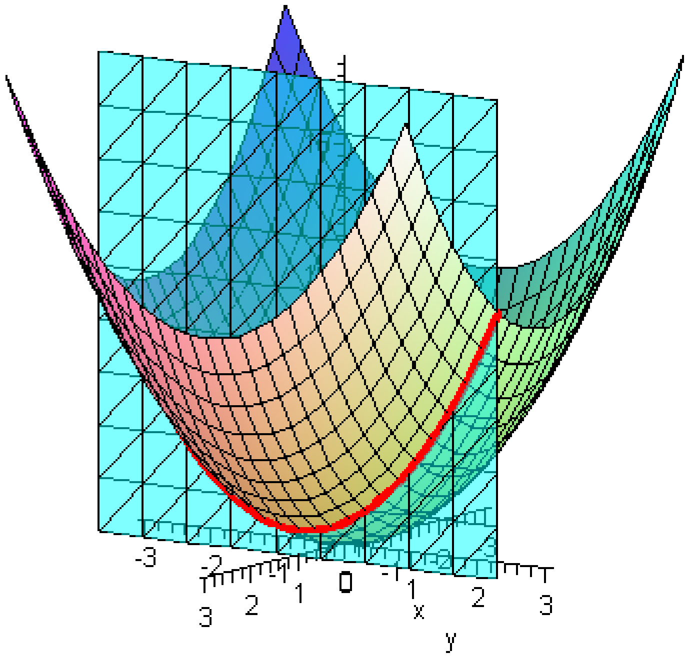
What are the appropriate parametric equations?

$$x = 1$$

$$y = t$$

$$z = 1^2 + t^2$$

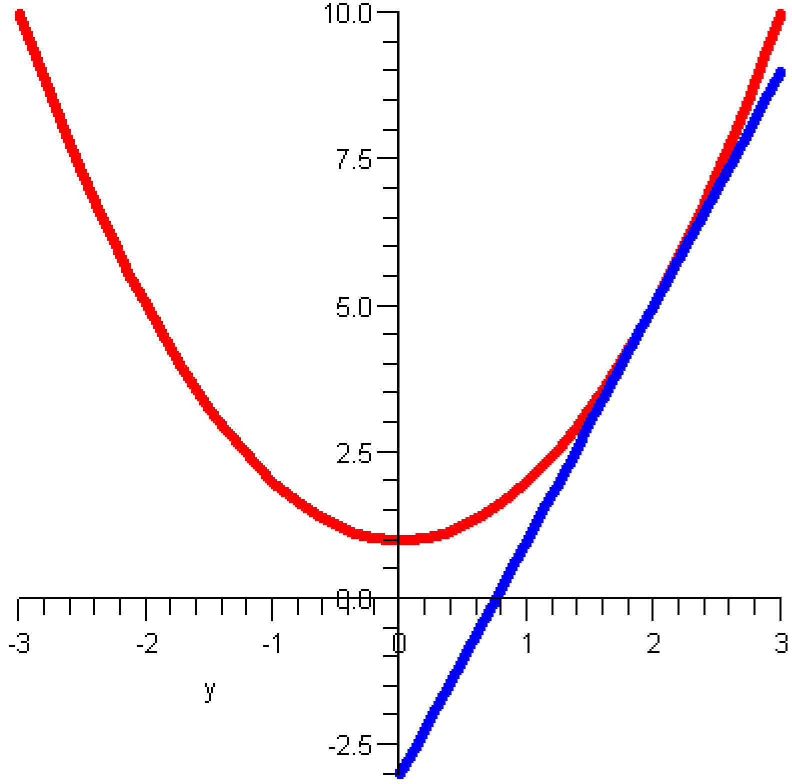
$$-3 \leq t \leq 3$$




```
spacecurve([1, t, 1
   + t^2], t = -3..3, color = red, thickness = 3, axes =
   normal);
```

Now let's go back to 2-dimensions and find the tangent line at the point (2,5).

$z(y) = y^2 + 1$

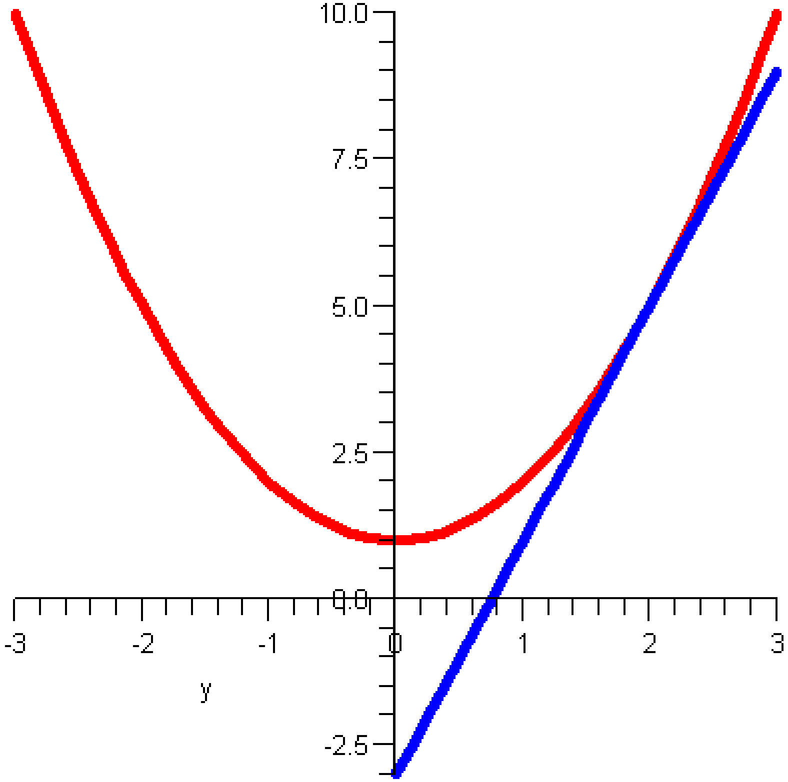


Now let's go back to 2-dimensions and find the tangent line at the point (2,5).

$$z(y) = y^2 + 1$$

$$z'(y) = 2y$$

$$z'(2) = 4 = slope$$



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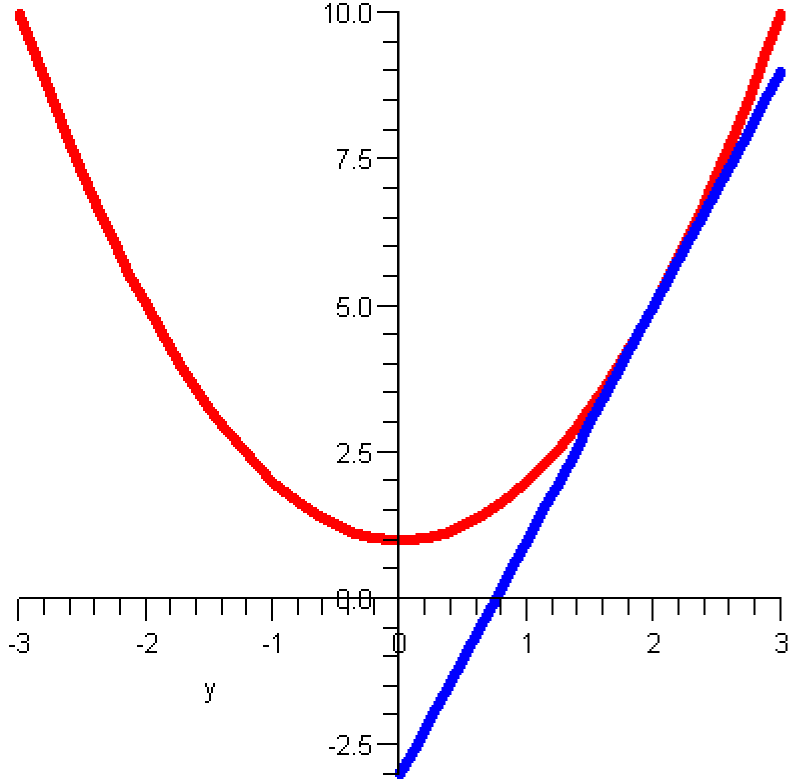
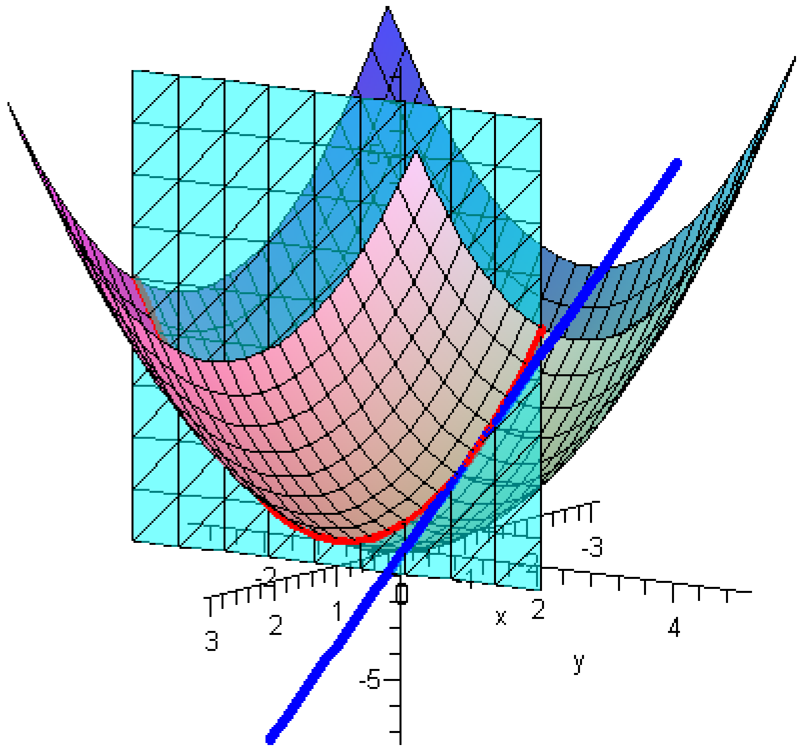


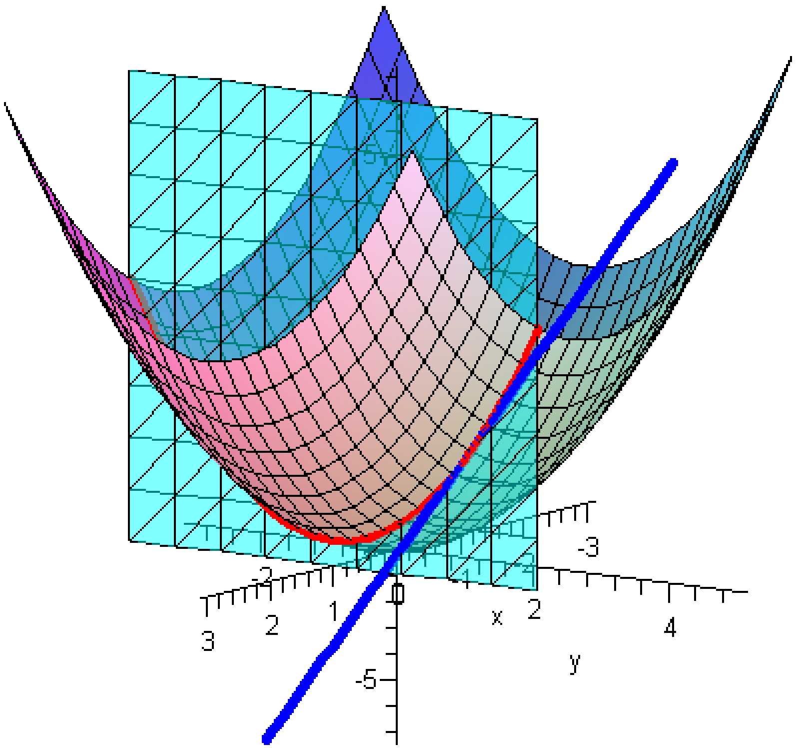
tangent line $= T = 4(y-2)+5 = 4y-3$

$\Rightarrow T = 4y - 3$

And finally, we can add this tangent line that lies in the plane $x = 1$ to our surface graph in 3-dimensions.



What could the parametric equations be for this tangent line in 3-dimensions?



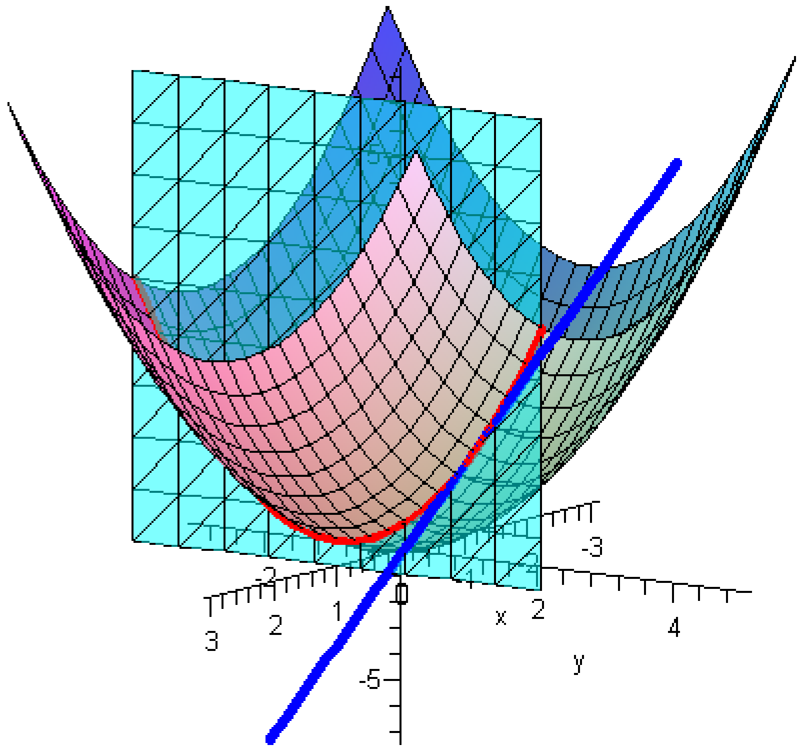
What could the parametric equations be for this tangent line in 3-dimensions?

$$x = 1$$

$$y = 2 + t$$

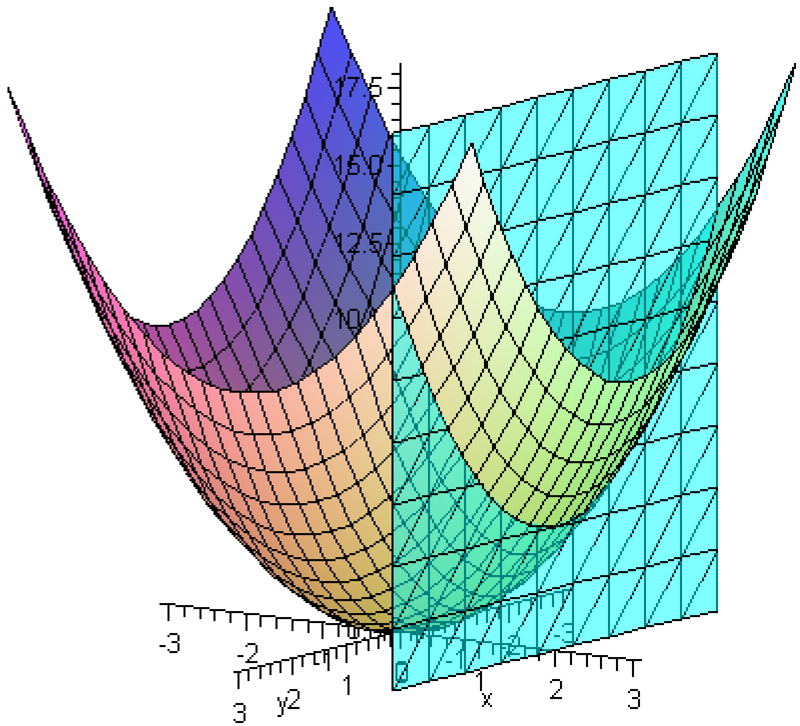
$$z = 5 + 4t$$

$$-3 \le t \le 3$$



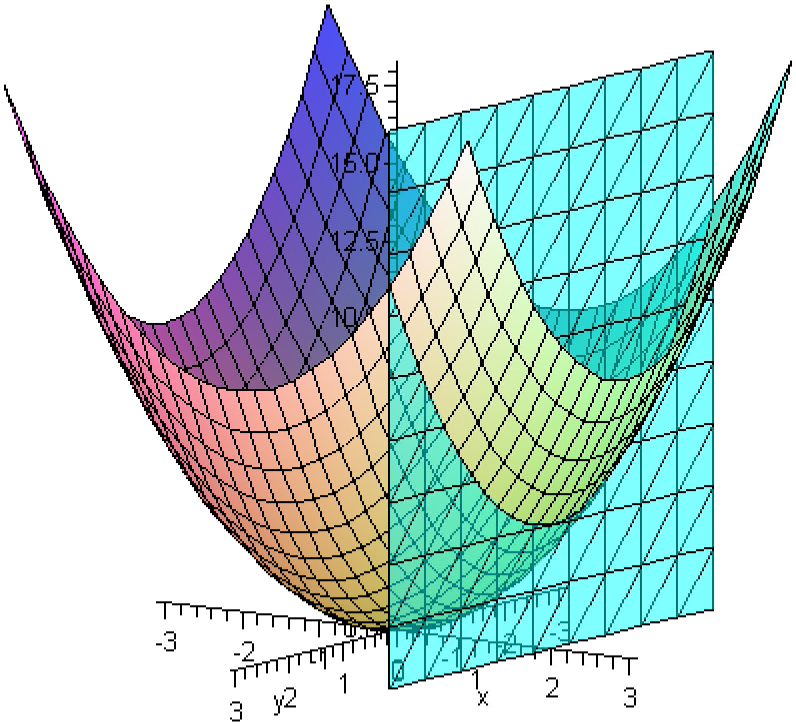
spacecurve([1, 2 + t, 5 + 4 t], t = −3 ..3, color = blue, thickness = 3);

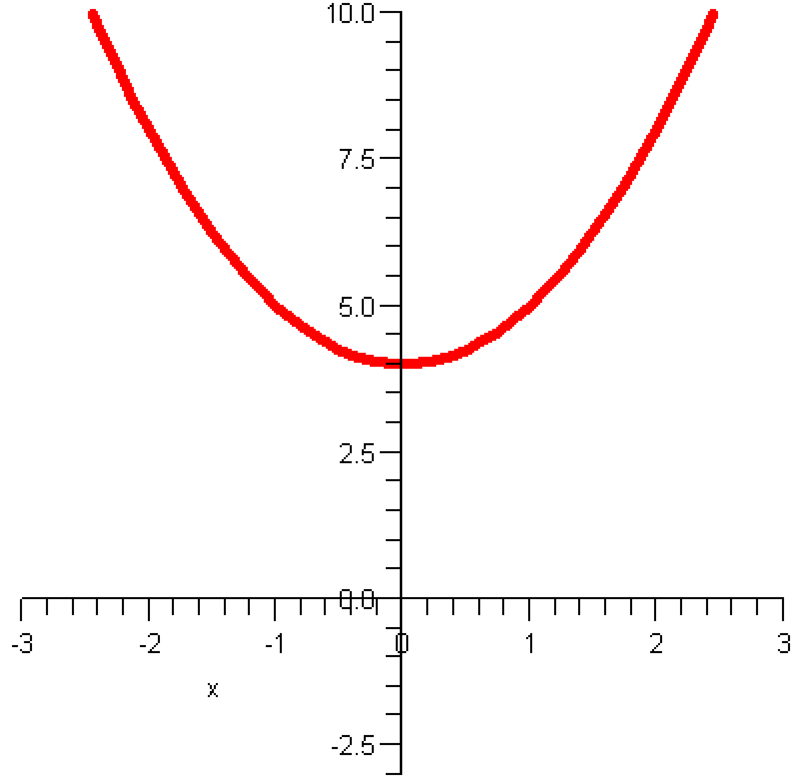
Now let's repeat everything starting this time with the intersection of our surface with the plane $y = 2$.



The curve of intersection in 2-dimensions is

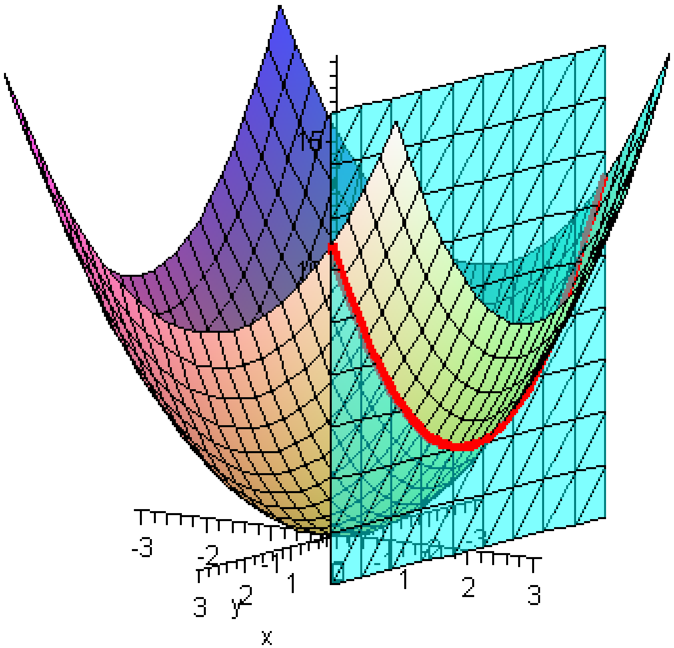
$$z = x^2 + 2^2 = x^2 + 4.$$





$plot(x^2 + 4,\ x = -3..3,\ view = [-3..3,\ -3..10], color = red,\ thickness = 3);$

What are the parametric equations needed to add this curve to our surface graph?



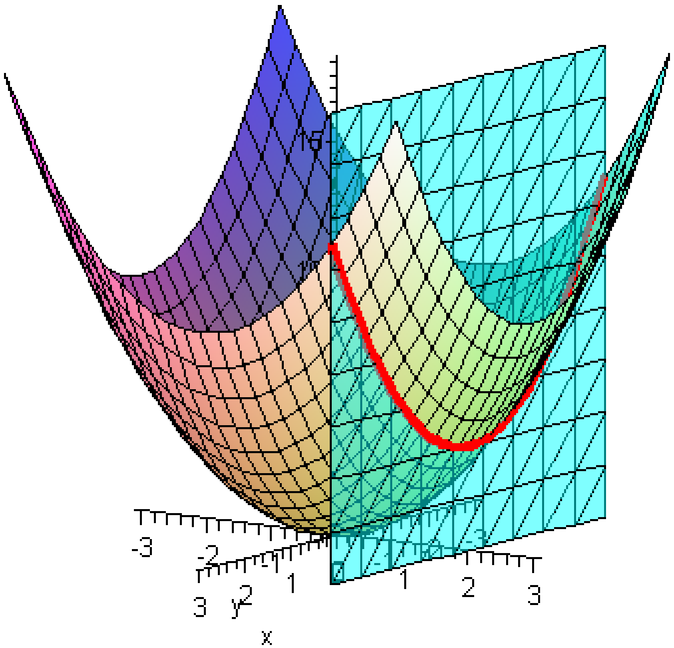
Here they are!

$x = t$

$y = 2$

$z = t^2 + 2^2$

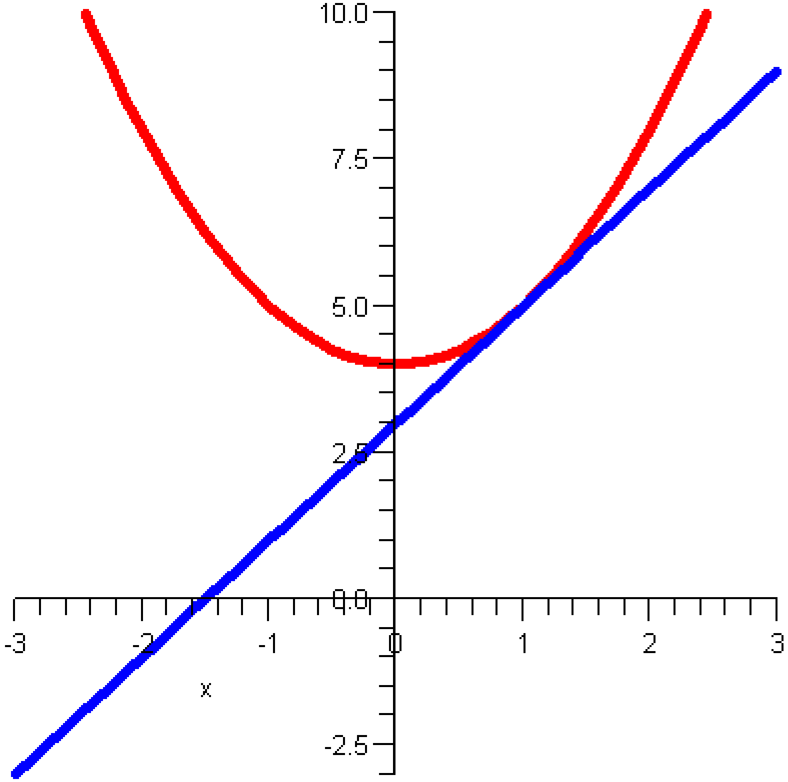
$-3 \leq t \leq 3$



$spacecurve([t, 2, t^2 + 4], t = -3..3, color = red, thickness = 3, axes = normal);$

Now we need to go back to 2-dimensions and find the tangent line at the point (1,5).

$z(x) = x^2 + 4$

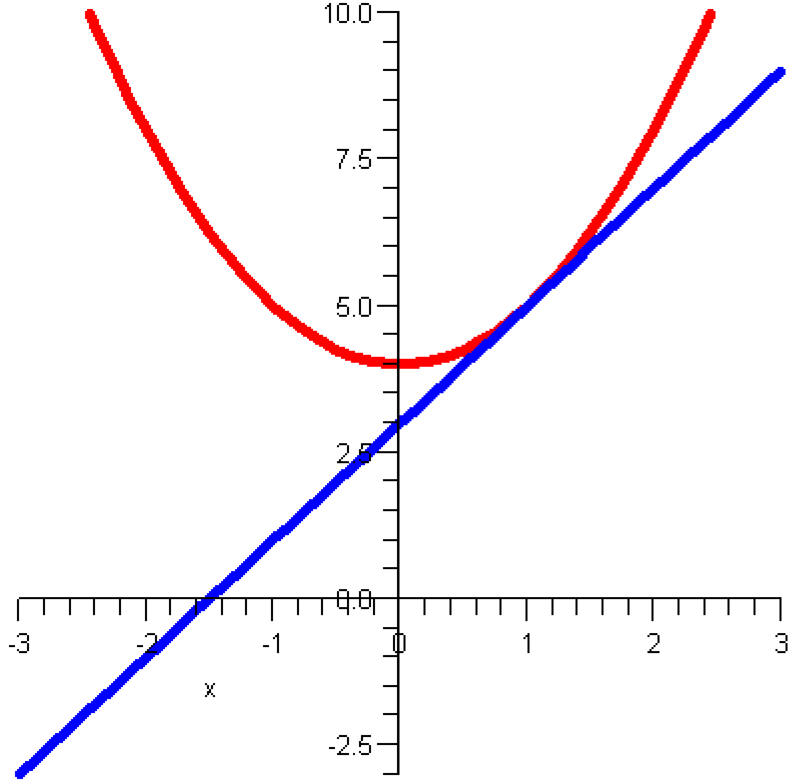


Now we need to go back to 2-dimensions and find the tangent line at the point (1,5).

$$z(x) = x^2 + 4$$

$$z'(x) = 2x$$

$$z'(1) = 2 = slope$$



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$$z(x) = x^2 + 4$$

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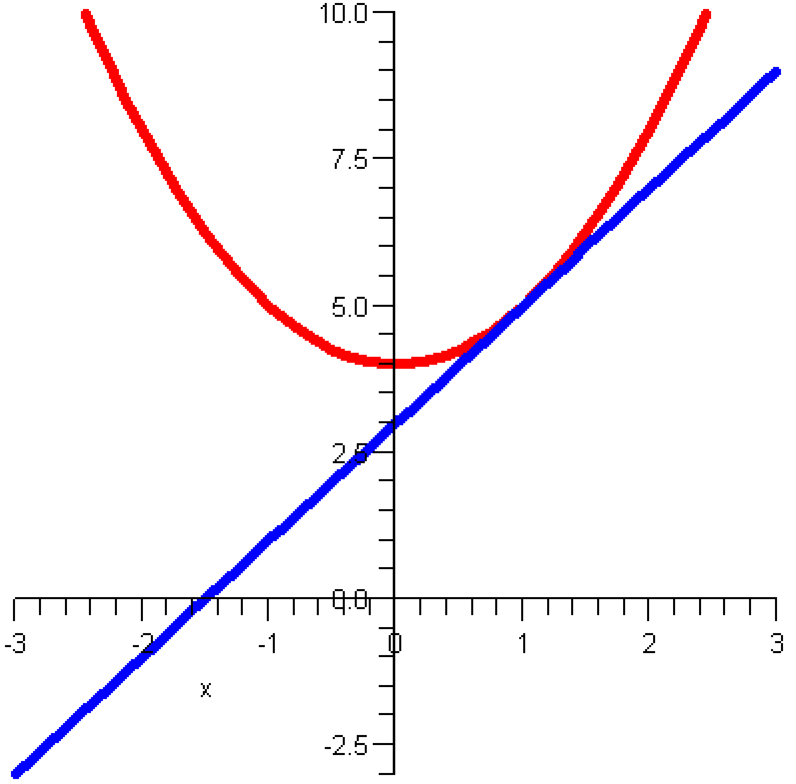
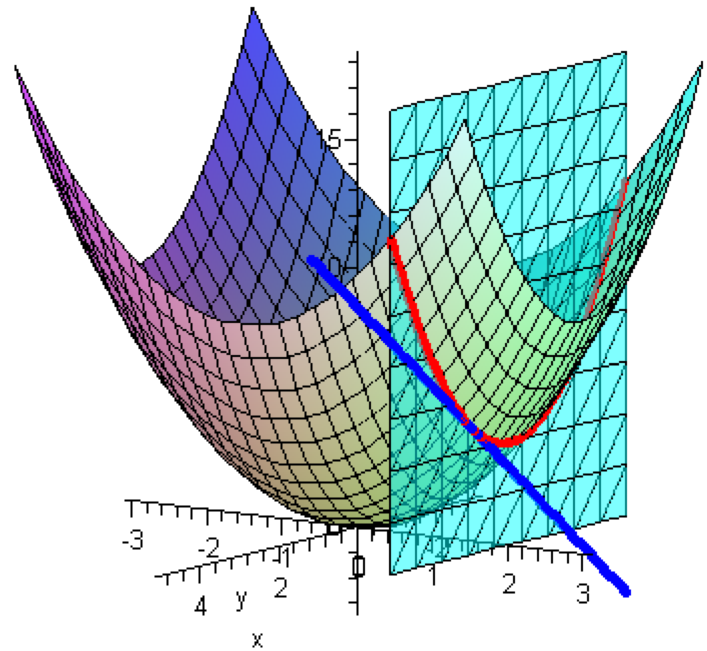


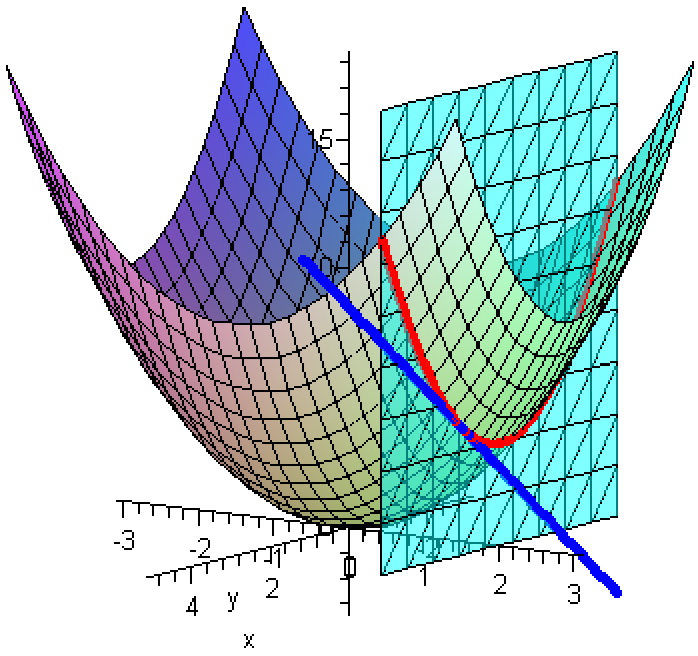
$$\text{tangent line } = T = 2(x-1)+5 = 2x+3$$

$$\Rightarrow T = 2x+3$$

And now we can add this tangent line that lies in the plane $y = 2$ to our surface graph in 3-dimensions.



What are the parametric equations for this tangent line in 3-dimensions?



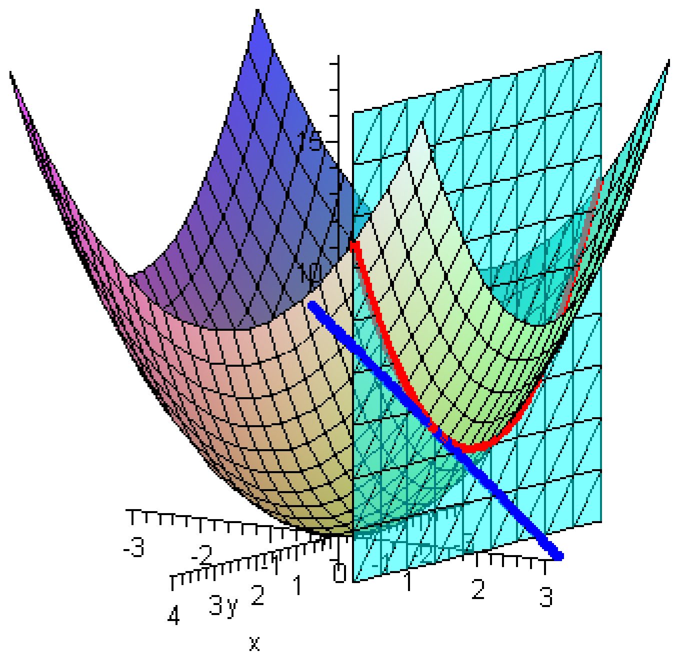
What are the parametric equations for this tangent line in 3-dimensions?

$$x = 1 + t$$

$$y = 2$$

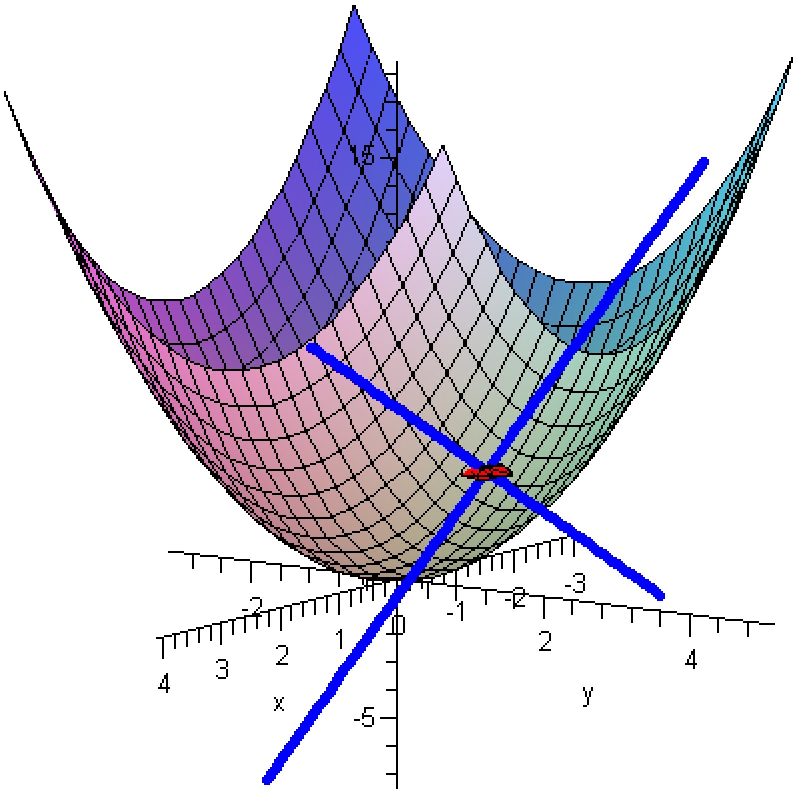
$$z = 5 + 2t$$

$$-3 \le t \le 3$$



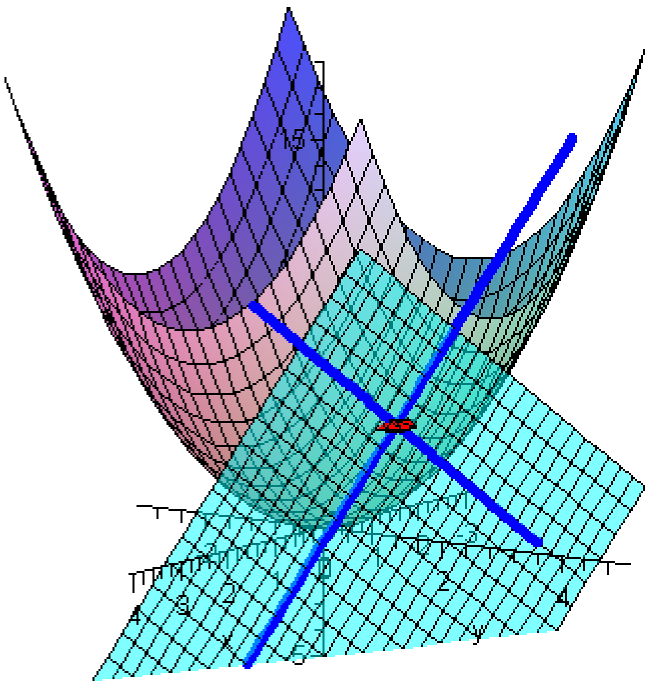
$spacecurve([1 + t, 2, 5 + 2t], t = -3..3, color = blue, thickness = 3);$

Now let's simultaneously look at both tangent lines at the point $(1, 2, 5)$ on our surface.



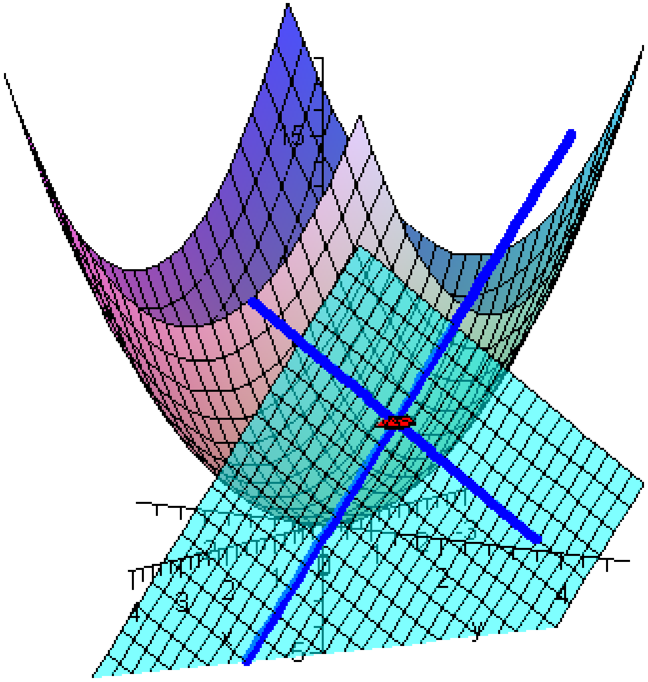
$implicitplot3d(0.1 = (x-1)^2 + (y-2)^2 + (z-5)^2, x = -3..3, y = -3..3, z = 0..8, color = red, axes = normal, orientation = [35, 80], numpoints = 15000);$

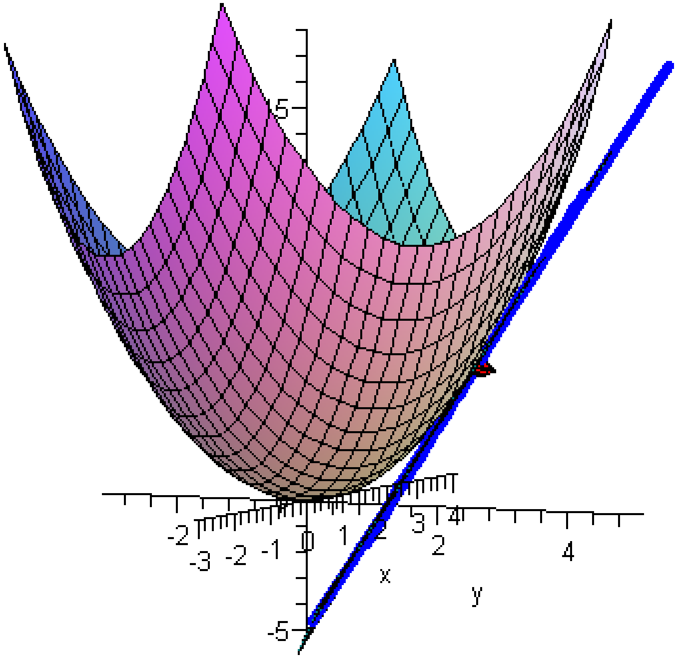
These two tangent lines define a tangent plane at the point $(1, 2, 5)$ on our surface.



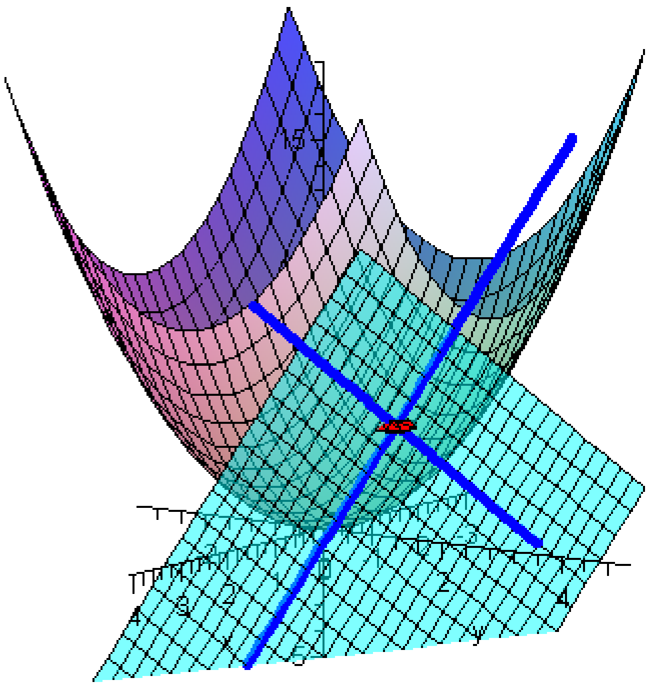
$$implicitplot3d(0.1 = (x-1)^2 + (y-2)^2 + (z-5)^2,\ x = -3..3,\ y = -3..3,\ z = 0..8,\ color = red,\ axes = normal,\ orientation = [35, 80],\ numpoints = 15000);$$

Can we find an equation for this tangent plane at the point (1,2,5)?



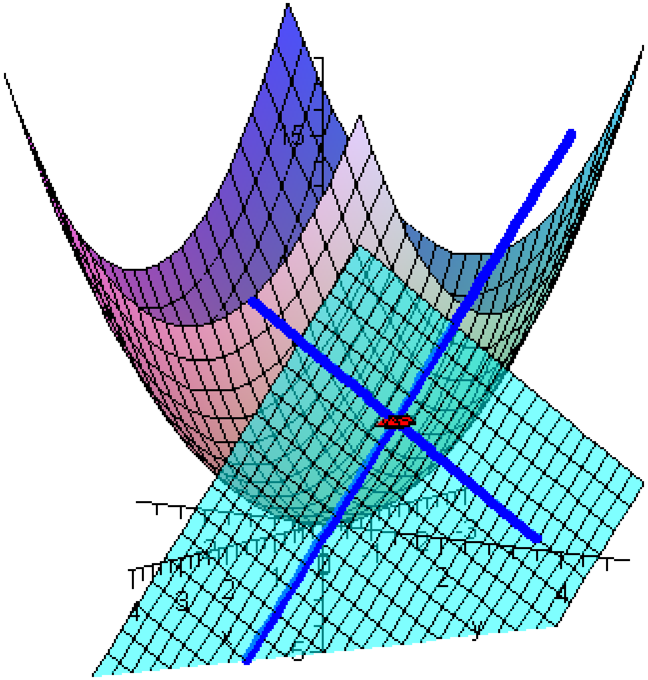


YES!



YES!

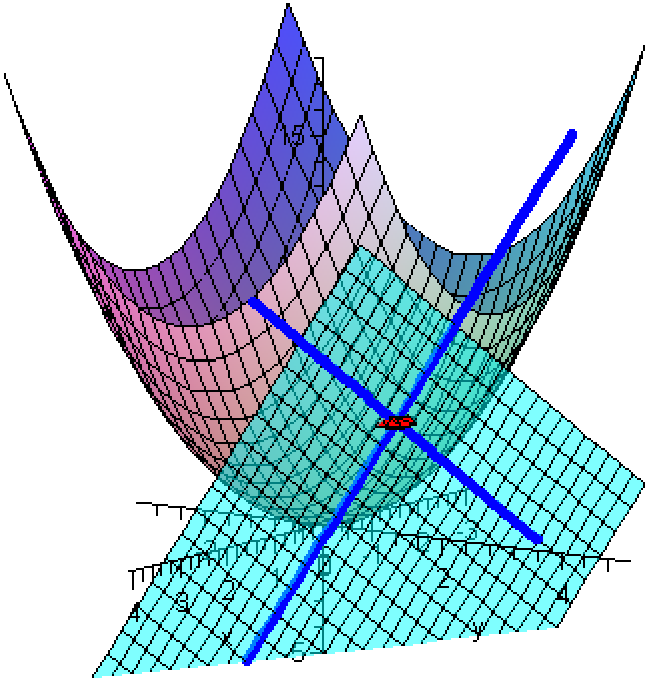
$$z = m_x x + m_y y + c$$



YES!

$$z = m_x x + m_y y + c$$

$$z = 2x + 4y + c$$

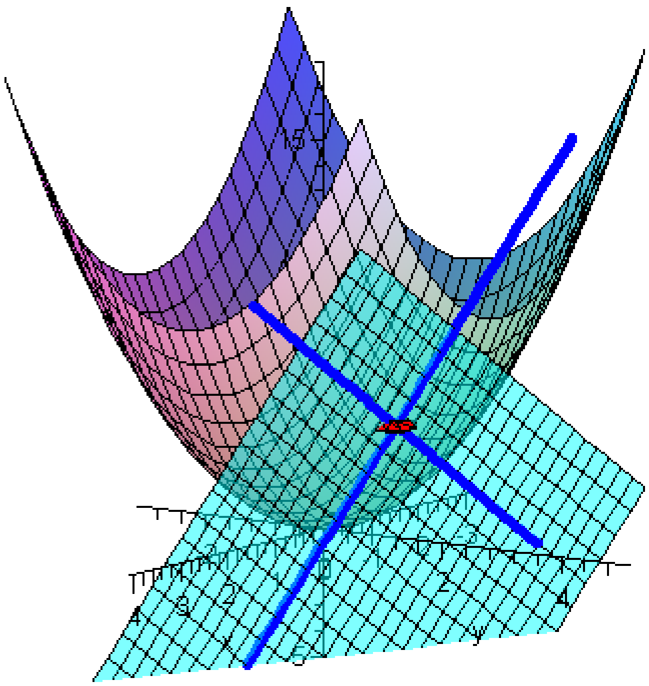


YES!

$$z = m_x x + m_y y + c$$

$$z = 2x + 4y + c$$

$$5 = 2(1) + 4(2) + c$$



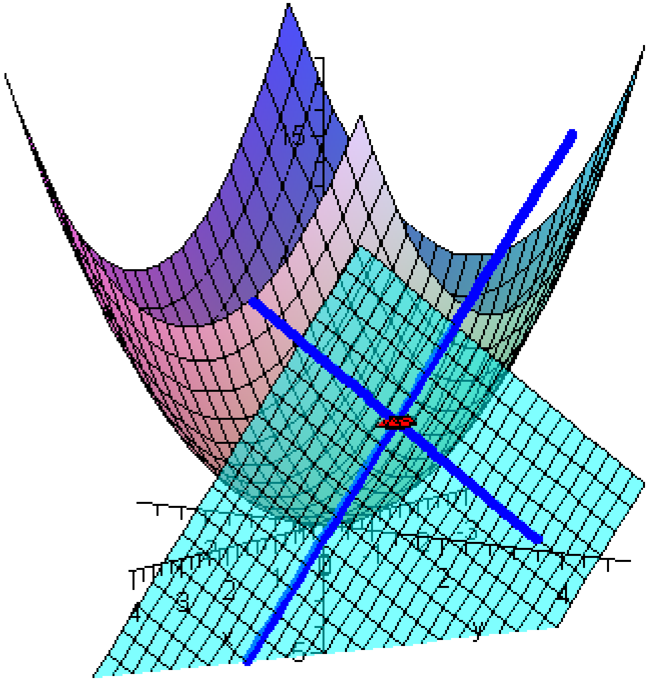
YES!

$$z = m_x x + m_y y + c$$

$$z = 2x + 4y + c$$

$$5 = 2(1) + 4(2) + c$$

$$c = -5$$



YES!

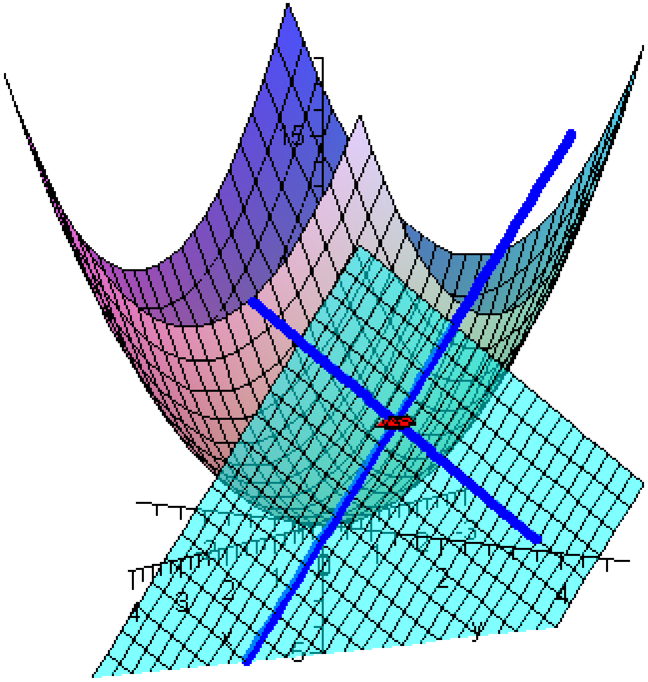
$$z = m_x x + m_y y + c$$

$$z = 2x + 4y + c$$

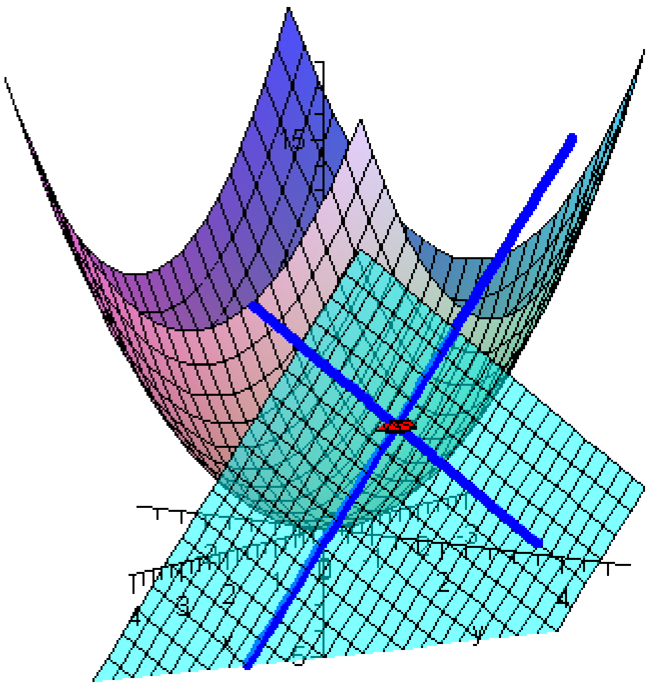
$$5 = 2(1) + 4(2) + c$$

$$c = -5$$

$$z = 2x + 4y - 5$$

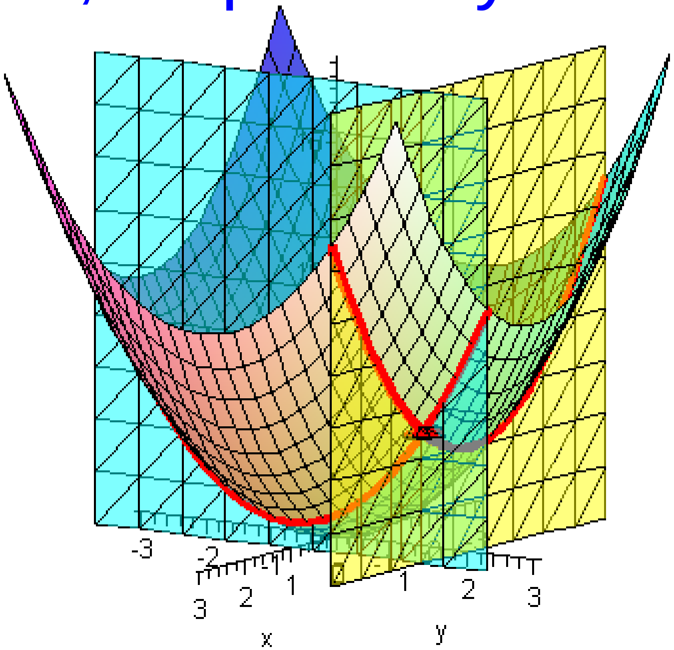


What have we learned?



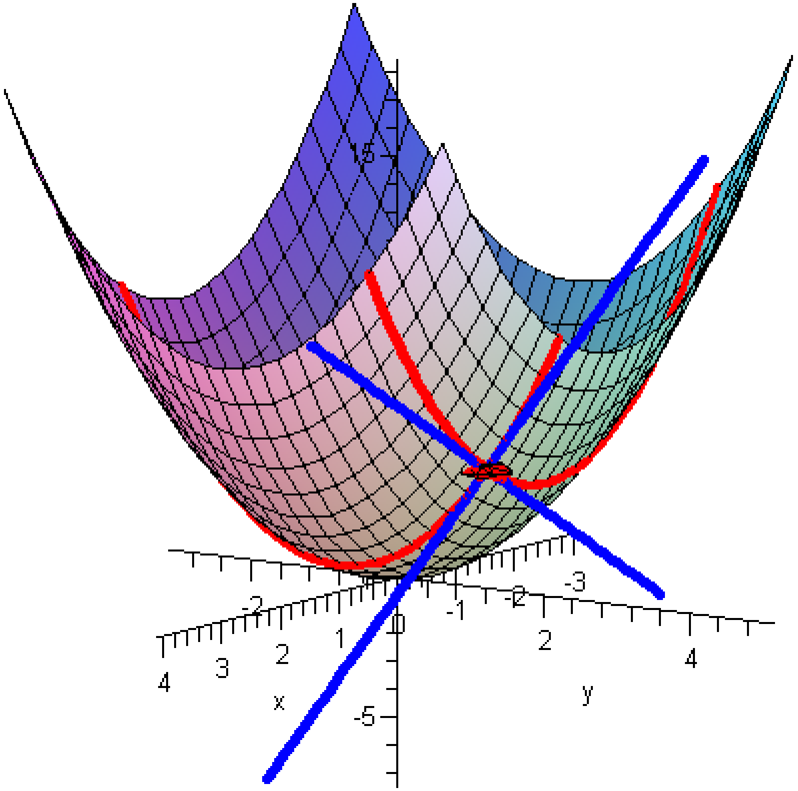
What have we learned?

- If we take a surface and a point *(a,b,c)* on the surface, then we can slice through that surface and the point with planes that are parallel to the *yz-* and *xz*-planes, respectively.



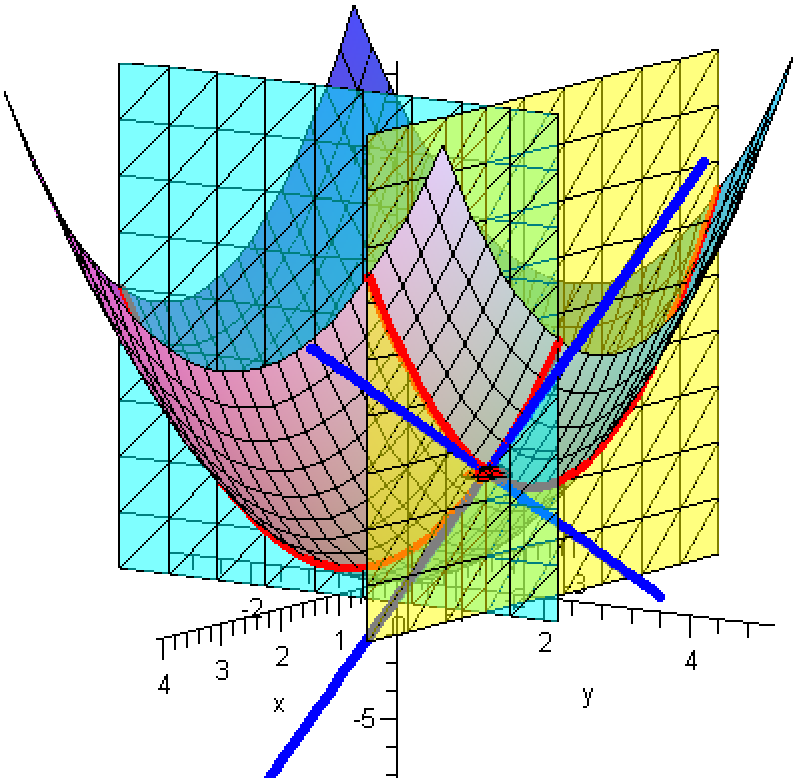
What have we learned?

- These slices produce curves of intersection and tangent lines at the point *(a,b,c)*.



What have we learned?

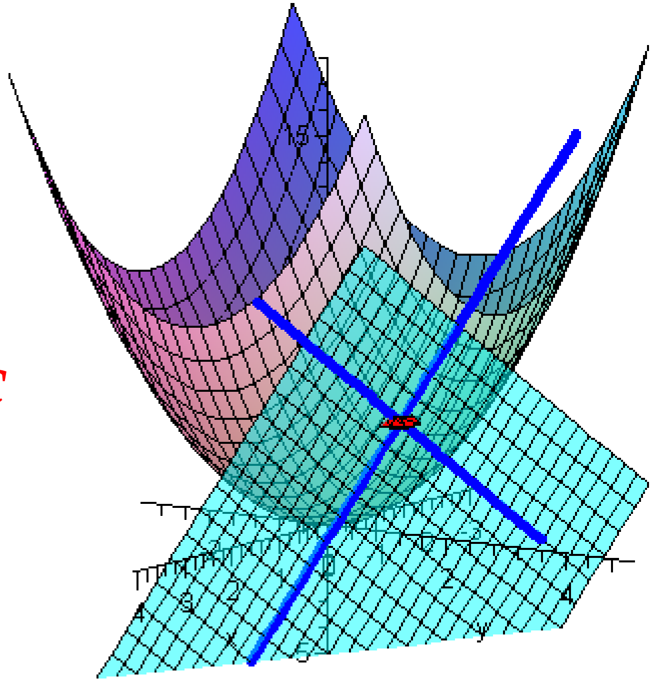
•One of these tangent lines is in the direction of the positive *x*-axis, and the other is in the direction of the positive *y*-axis.



What have we learned?

- These tangent lines can, furthermore, be used to construct a plane that is tangent to the surface at *(a,b,c)*.

$$z = m_x x + m_y y + c$$



And that's it!

$$z = m_x x + m_y y + c$$

$$P = (1, 2, 5)$$

$$z = 2x + 4y - 5$$