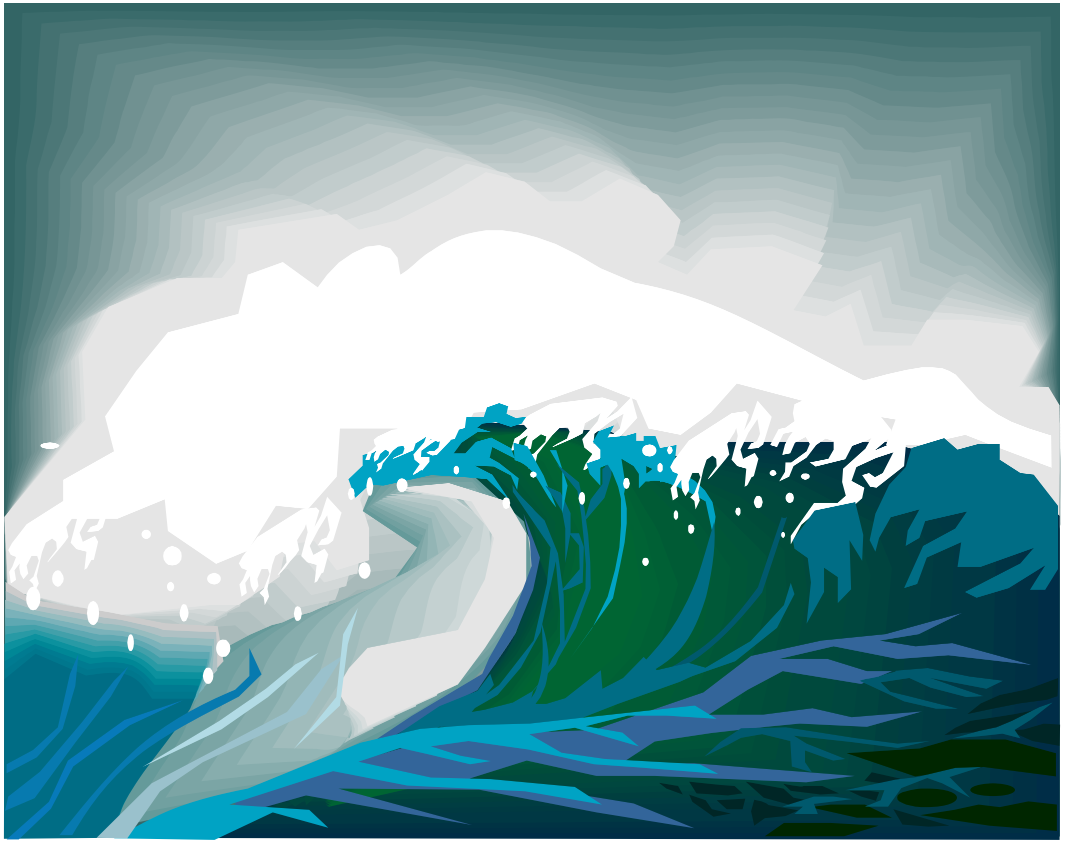
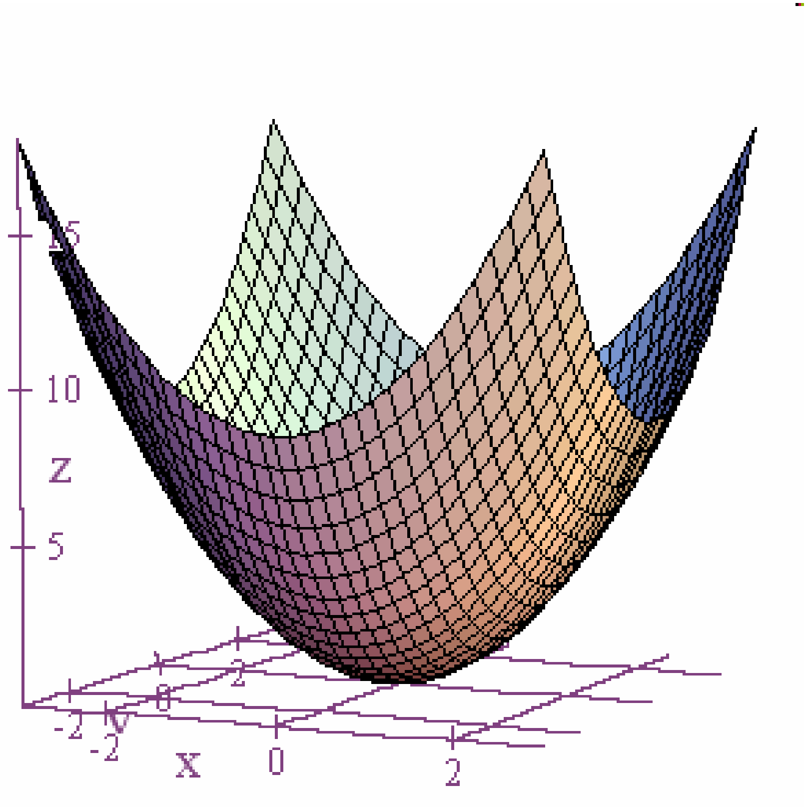
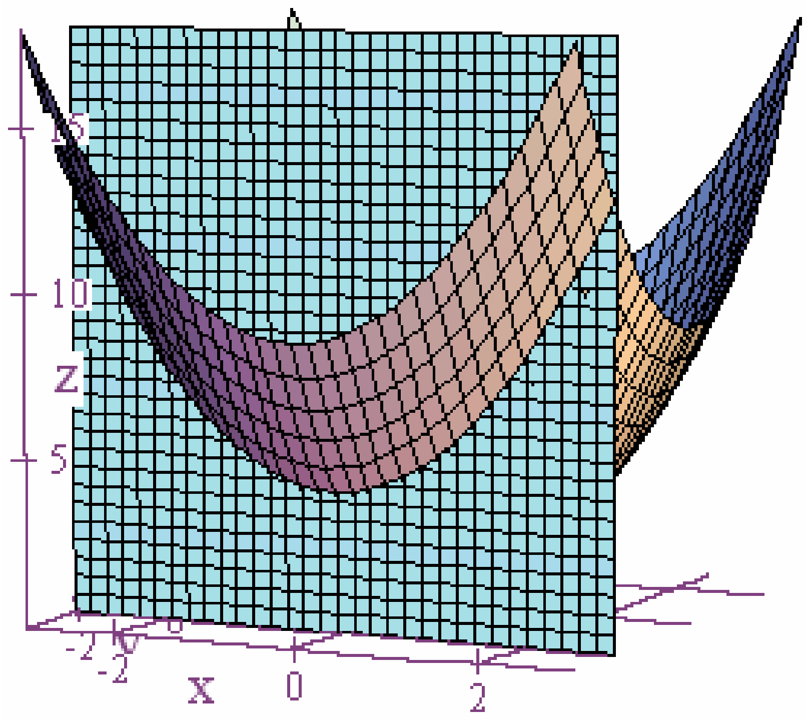
PARTIAL DERIVATIVES



Consider the surface $z = f(x, y) = x^2 + y^2$



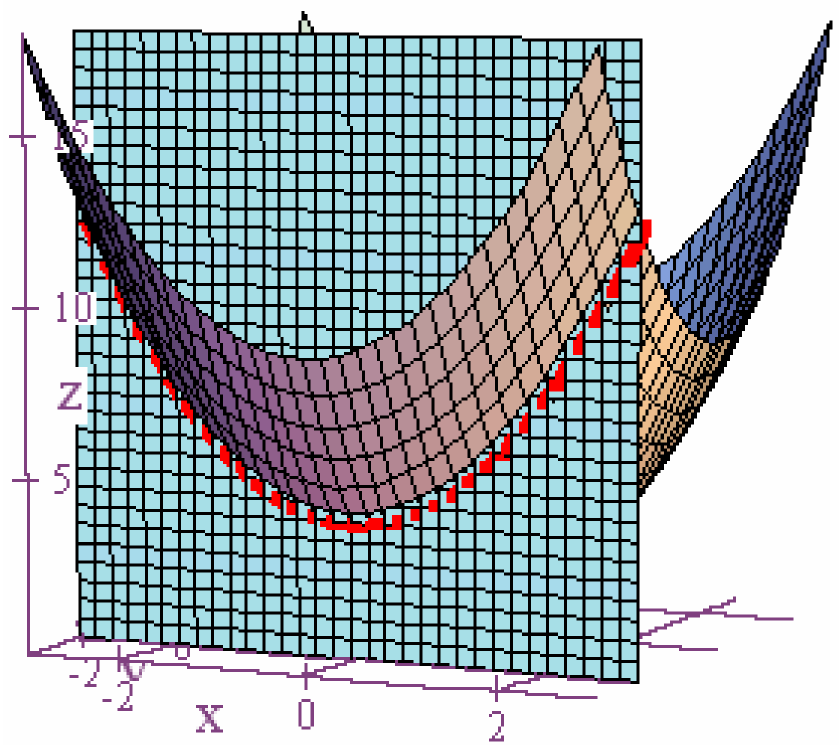
We can slice through this surface with the plane *y*=*-2*.



$$z = f(x, y) = x^2 + y^2$$

When we do this, the the curve of intersection is desribed by the equations:

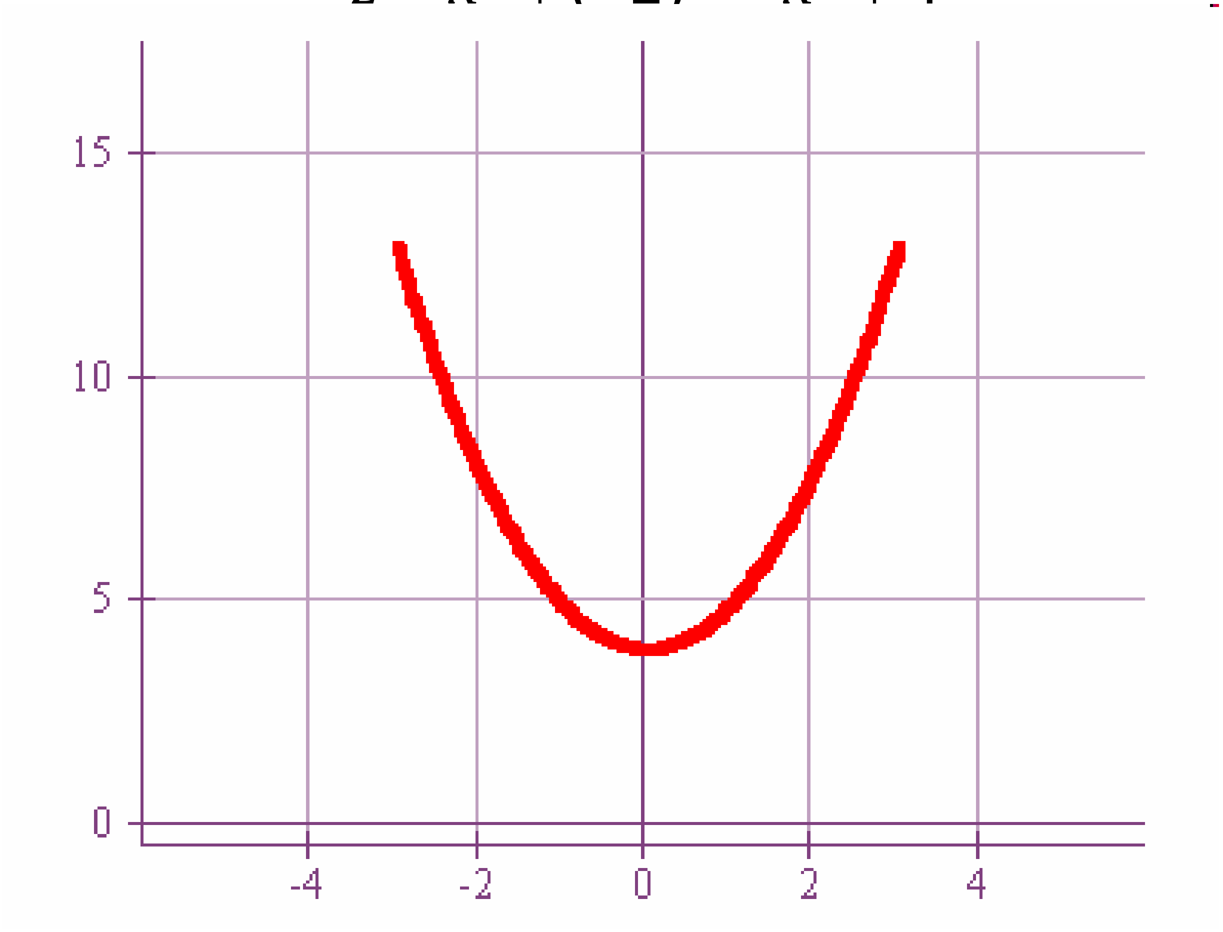
$$z = x^2 + (-2)^2 = x^2 + 4, \;\; y = -2$$



$$z = f(x, y) = x^2 + y^2$$

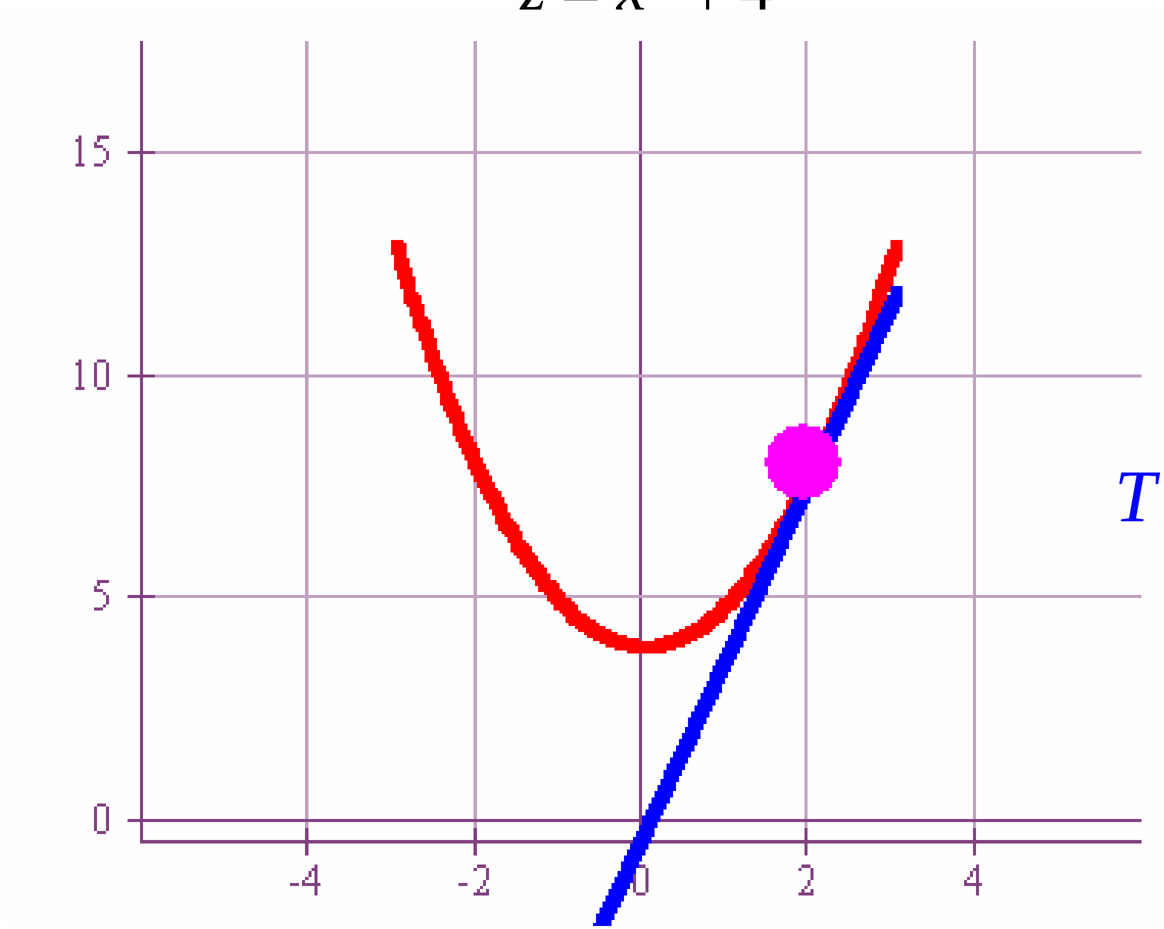
We can graph this equation in 2-dimensions.

$$z = x^2 + (-2)^2 = x^2 + 4$$



And if we take a point such as *(2,8)* on the graph, then we can use derivatives to find the tangent line.

$$z = x^2 + 4$$



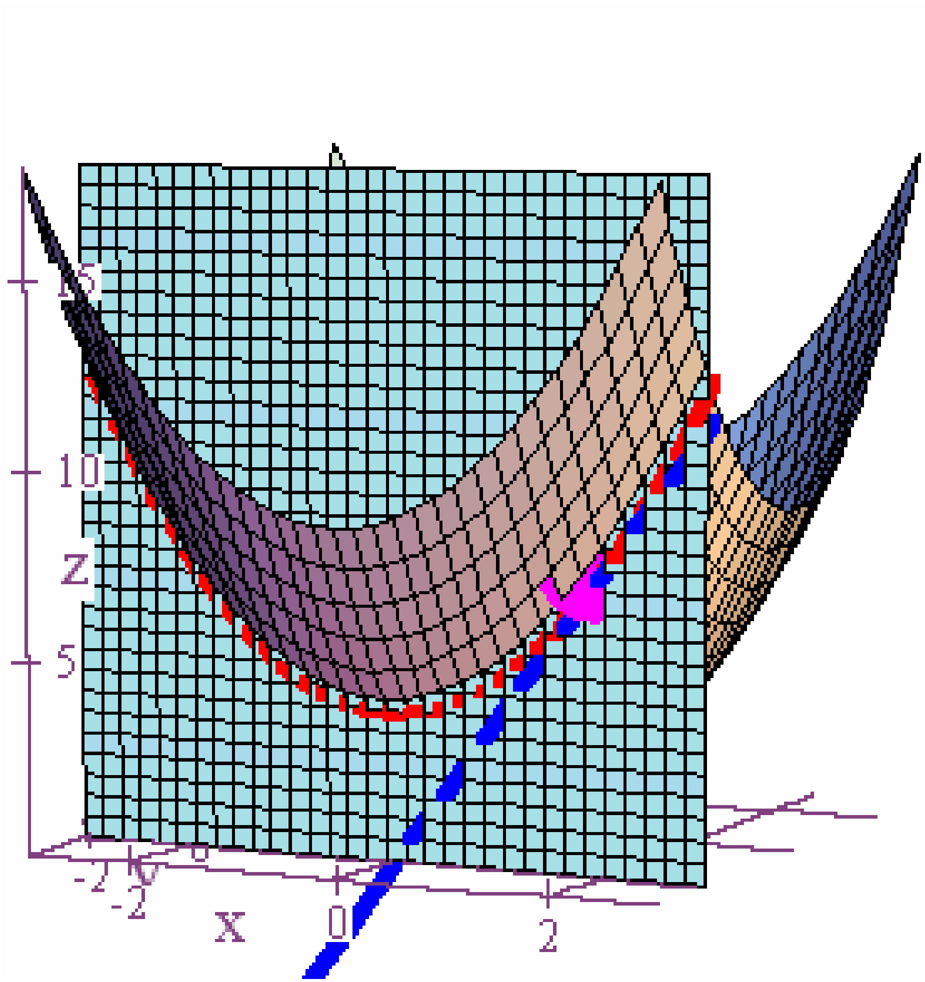
$$z'(x) = 2x$$

$$z'(2) = 4$$

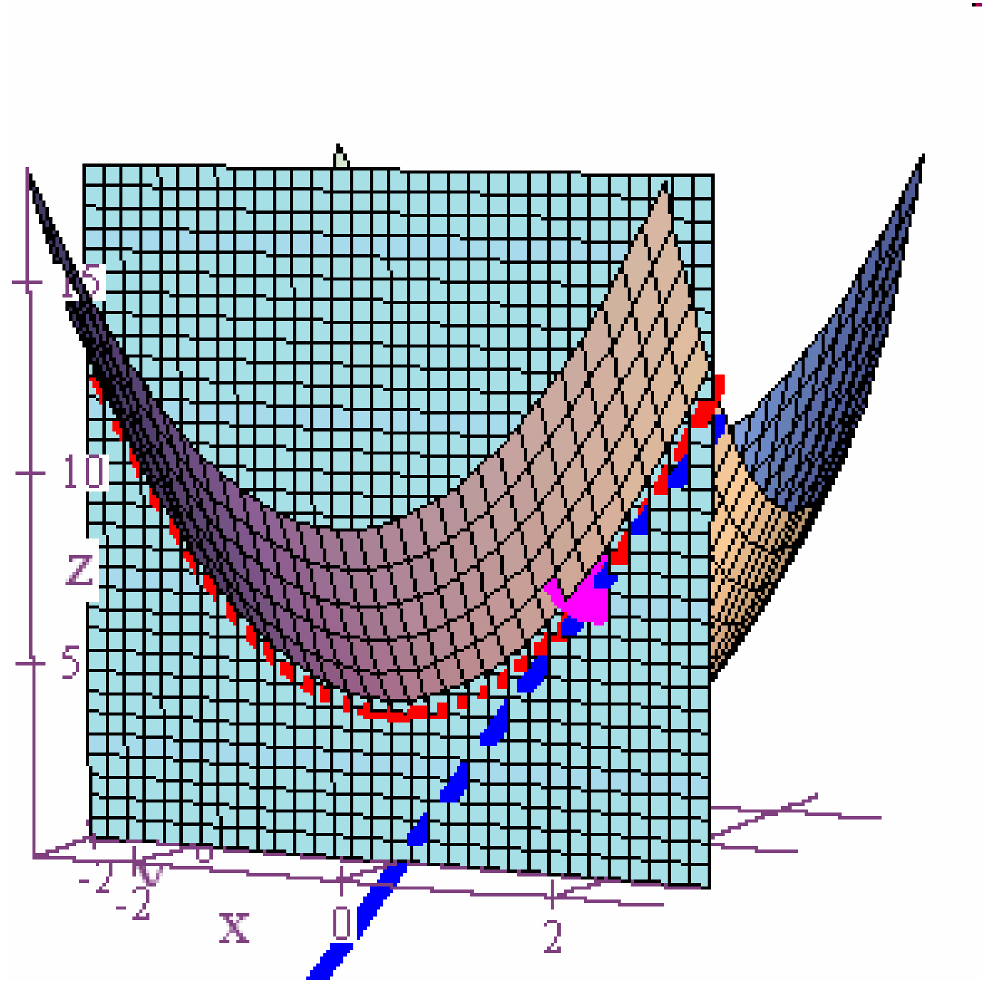
$$T = 4(x-2) + 8$$

$$= 4x$$

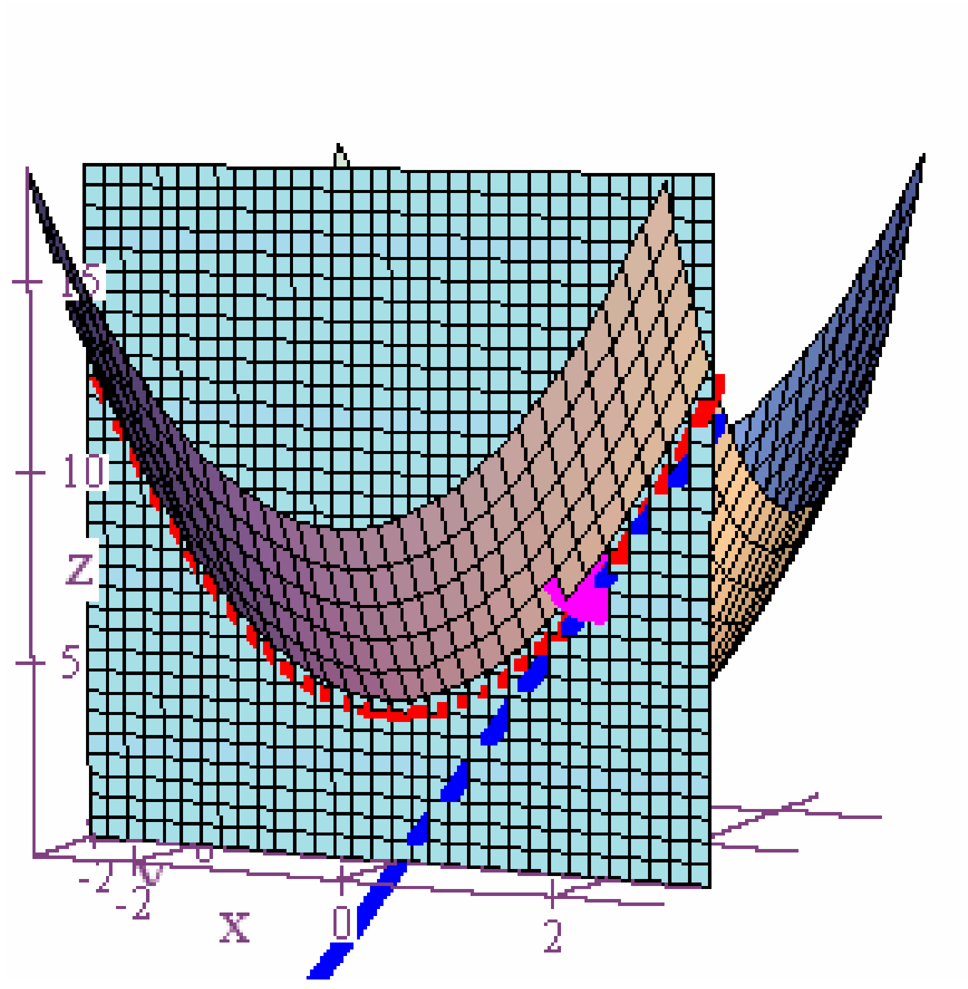
If we add the point and the tangent line back to our surface plot, then it looks like this:



Here’s what it all means.

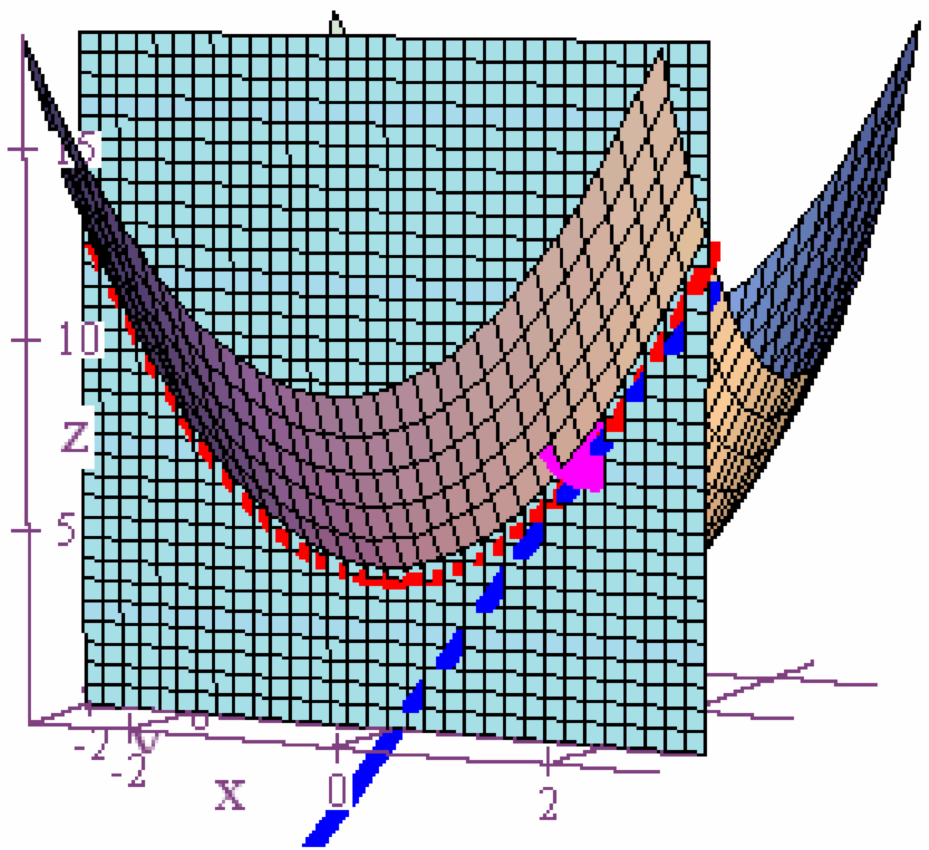


We start with a surface $z = f(x, y) = x^2 + y^2$



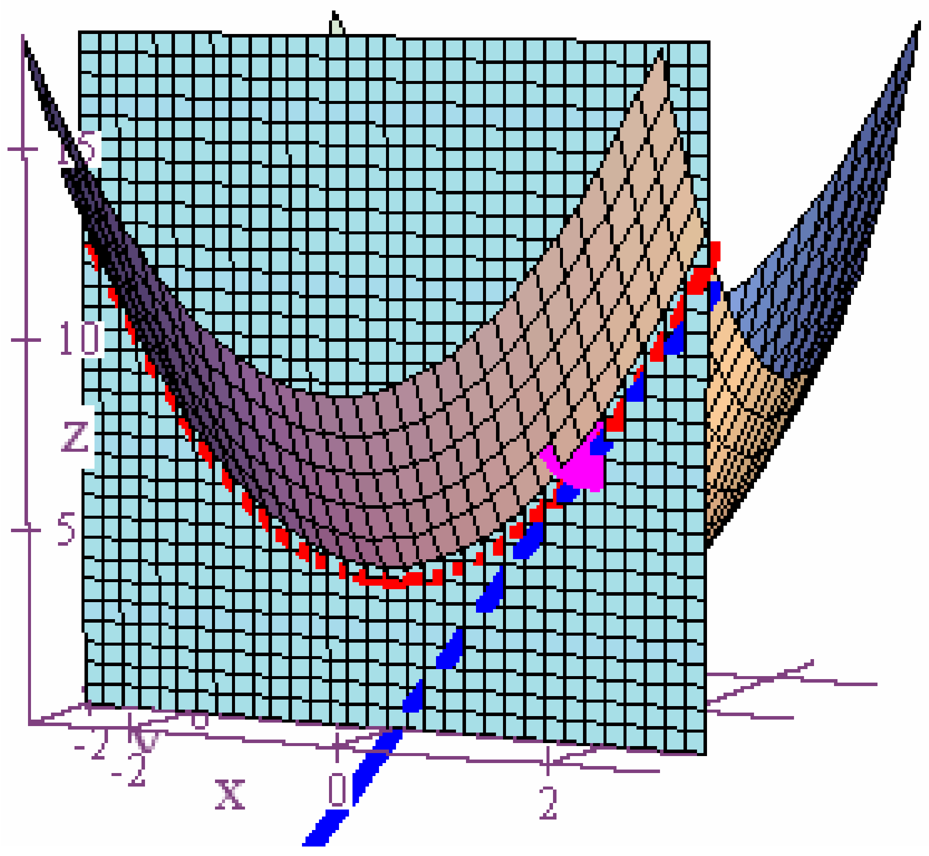
On this surface is a point *(2,-2,8)*.

$$z = f(x,y) = x^2 + y^2$$



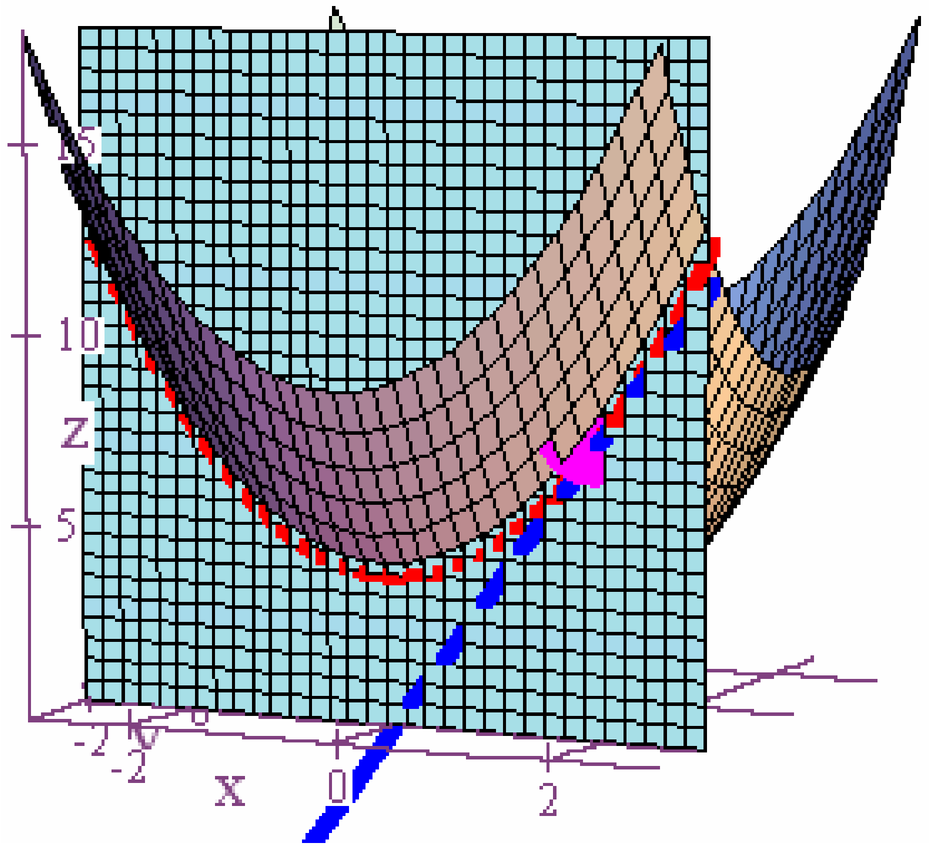
If we slice through the surface with the plane *y*=-2, we get a curve of intersection with our surface.

$$z = f(x, y) = x^2 + y^2$$



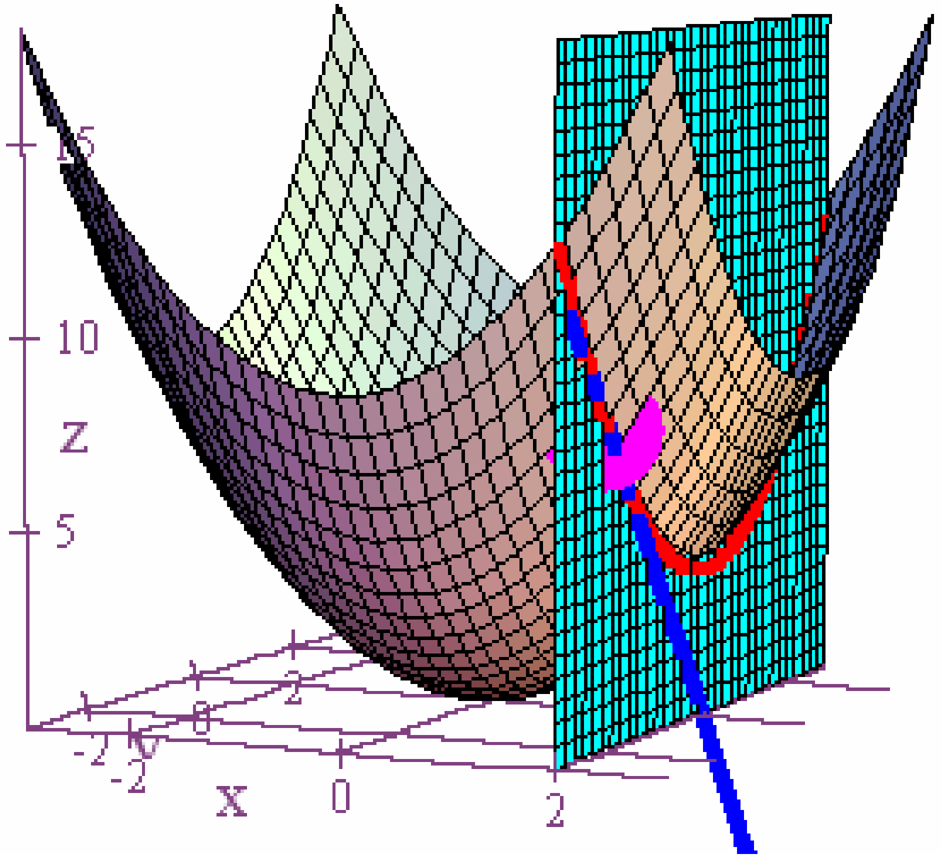
The blue line is the line that is tangent to the surface at *(2,-2,8)* and that lies in the plane *y=-2.*

$$z = f(x, y) = x^2 + y^2$$



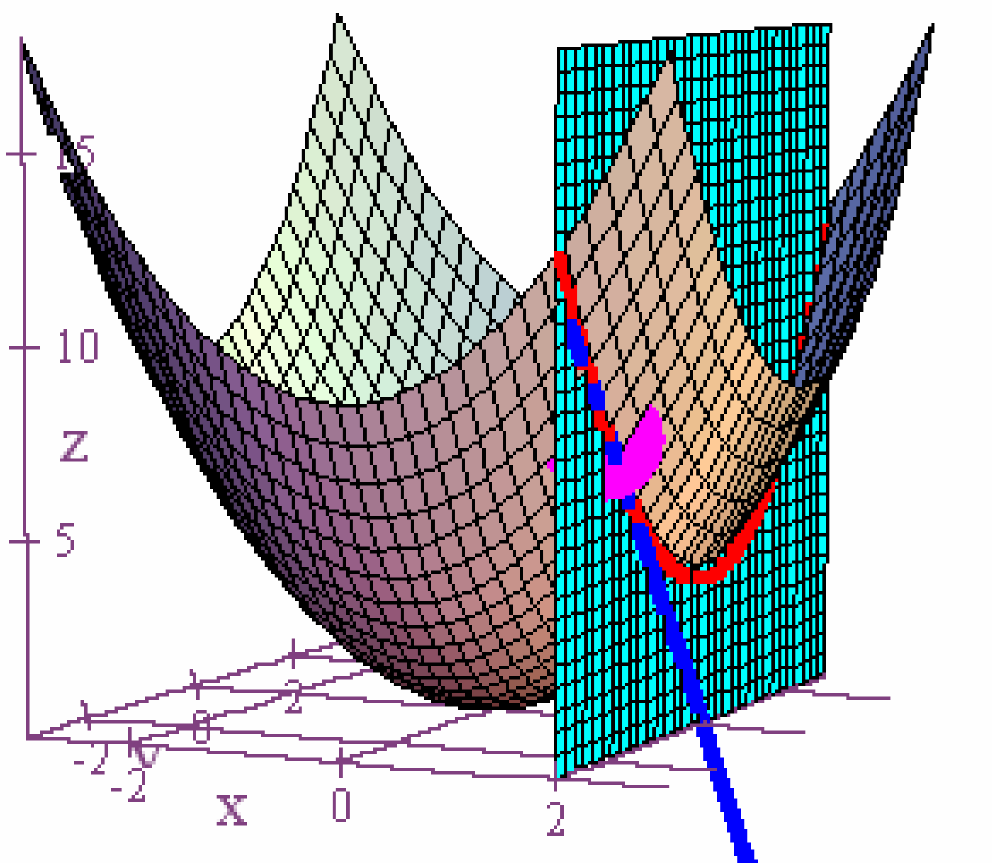
We can do something similar by slicing through with the plane *x=2*.

$$z = f(x, y) = x^2 + y^2$$

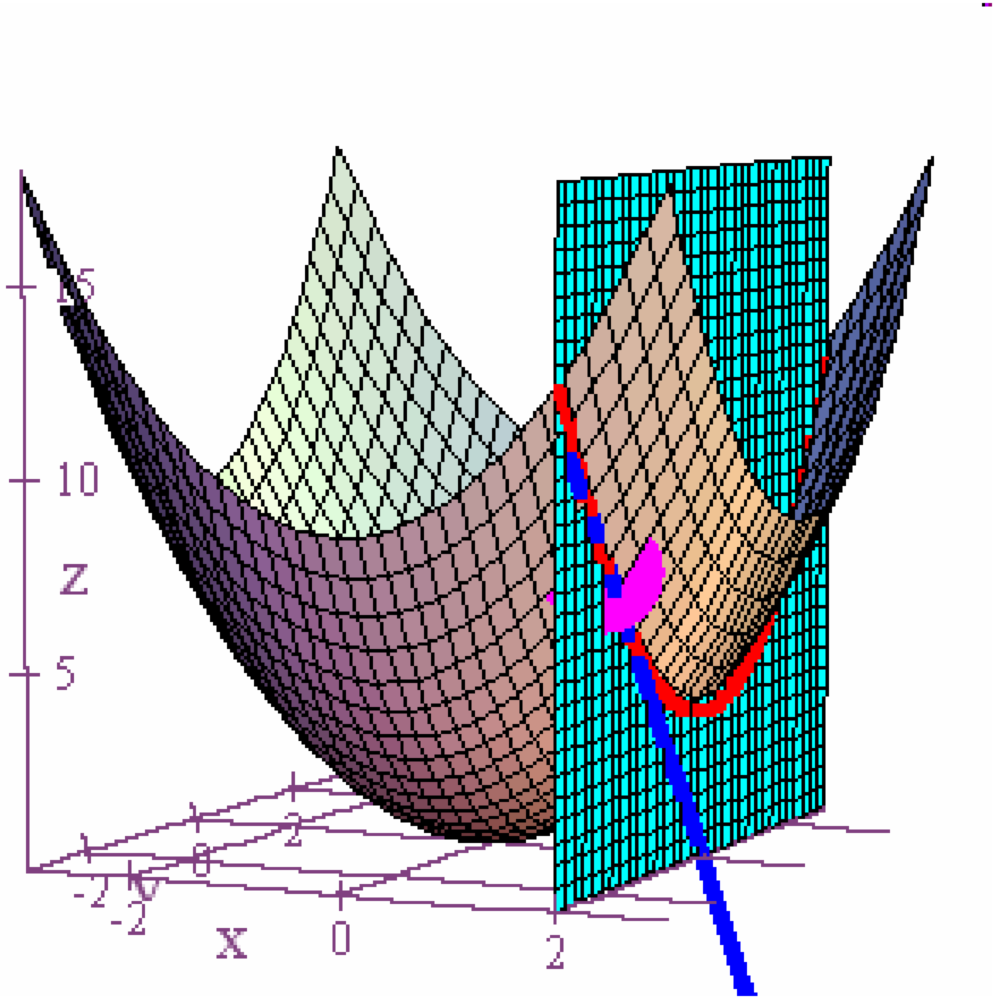


This time, the curve of intersection is:

$$z = 2^2 + y^2 = 4 + y^2,\ \ x = 2$$



The slope of the tangent line at *(2,-2,8)* and in the plane *x=2* can be found using derivatives.



$$z = 4 + y^2$$

$$z'(y) = 2y$$

$$z'(-2) = -4$$

These examples illustrate several points.

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1. *A point on a surface can have an infinite number of tangent lines, each one pointing in a different direction.*

2. *Two of these tangent lines can be found by fixing either the x-coordinate or the y-coordinate, and then taking the derivative in order to find the slope.*

In practice, instead of actually fixing an x-value or y-value, we just pretend that we have and then differentiate with respect to the other variable.

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And we use a slightly different notation.

<u>Definition:</u> If *z=f(x,y)*, then the partial derivative of *z* with respect to *x* is:

$$z_x = \frac{\partial z}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

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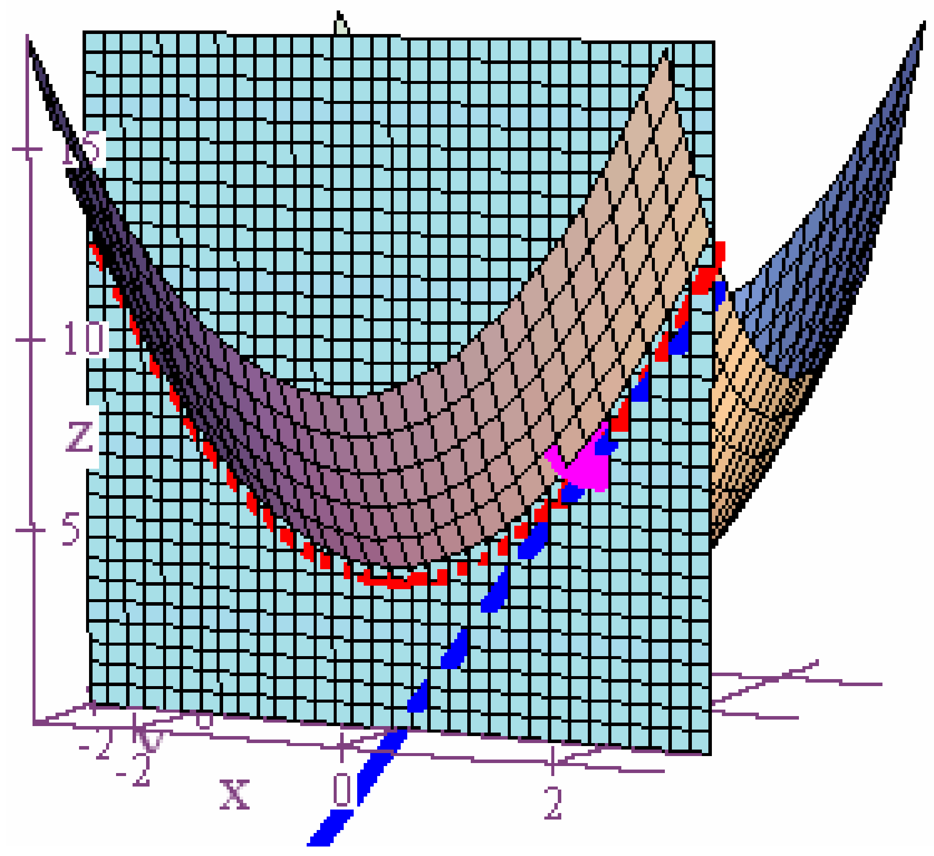
$$z = x^2 + y^2$$

$$z_y = \frac{\partial z}{\partial y} = 2y$$

To reiterate, the partial derivative of *z* with respect to *x* can be used to find the slope of a tangent line to a point on a curve of intersection obtained by fixing *y*.



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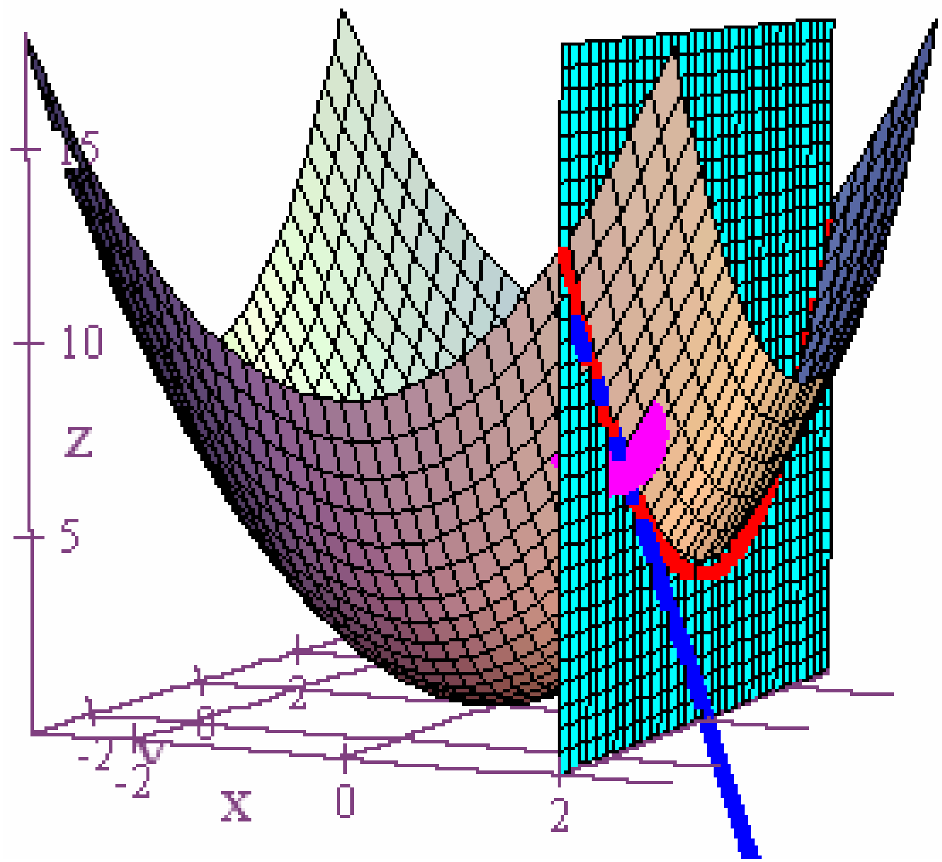

<u>Practice:</u> For each of the following functions, find $\frac{\partial z}{\partial x}$ & $\frac{\partial z}{\partial y}$.

1. $z = x^3 y^2$
2. $z = \sqrt{x^2 + y^2}$
3. $z = \ln(xy)$
4. $z = \sin\left(\frac{x}{y}\right)$
5. $z = x^y$