The Gradient Vector

If $z = f(x, y)$, then the gradient of z is

$$grad(z) = \nabla z = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j}.$$

If $w = f(x, y, z)$, then the gradient of w is

$$grad(w) = \nabla w = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}.$$

Note: ∇z is also read as "del z."

$$z = x^2 + y^2$$

$$P = (1, 2, 5)$$

$$\nabla z = 2x\hat{i} + 2y\hat{j}$$

$$\nabla z(1, 2) = 2\hat{i} + 4\hat{j}$$

$$w = x^2 - y^3 + z^2$$

$$Q = (1, 1, 2, 4)$$

$$\nabla w = 2x\hat{i} - 3y^2\hat{j} + 2z\hat{k}$$

$$\nabla w(1, 1, 2) = 2\hat{i} - 3\hat{j} + 4\hat{k}$$