# FUNCTIONS OF SEVERAL VARIABLES

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As usual, a set of specific values for the inputs always determines a specific value for the output.

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2. $$Perimeter = 2L + 2W$$

3. $$A = P\left(1+\frac{r}{n}\right)^{nt}$$

4. $$z = f(x,y) = x^2 + y^2$$

A function of several variables may be expressed in several different ways.

Verbally:

“The output is the sum of the squares of the two inputs.”

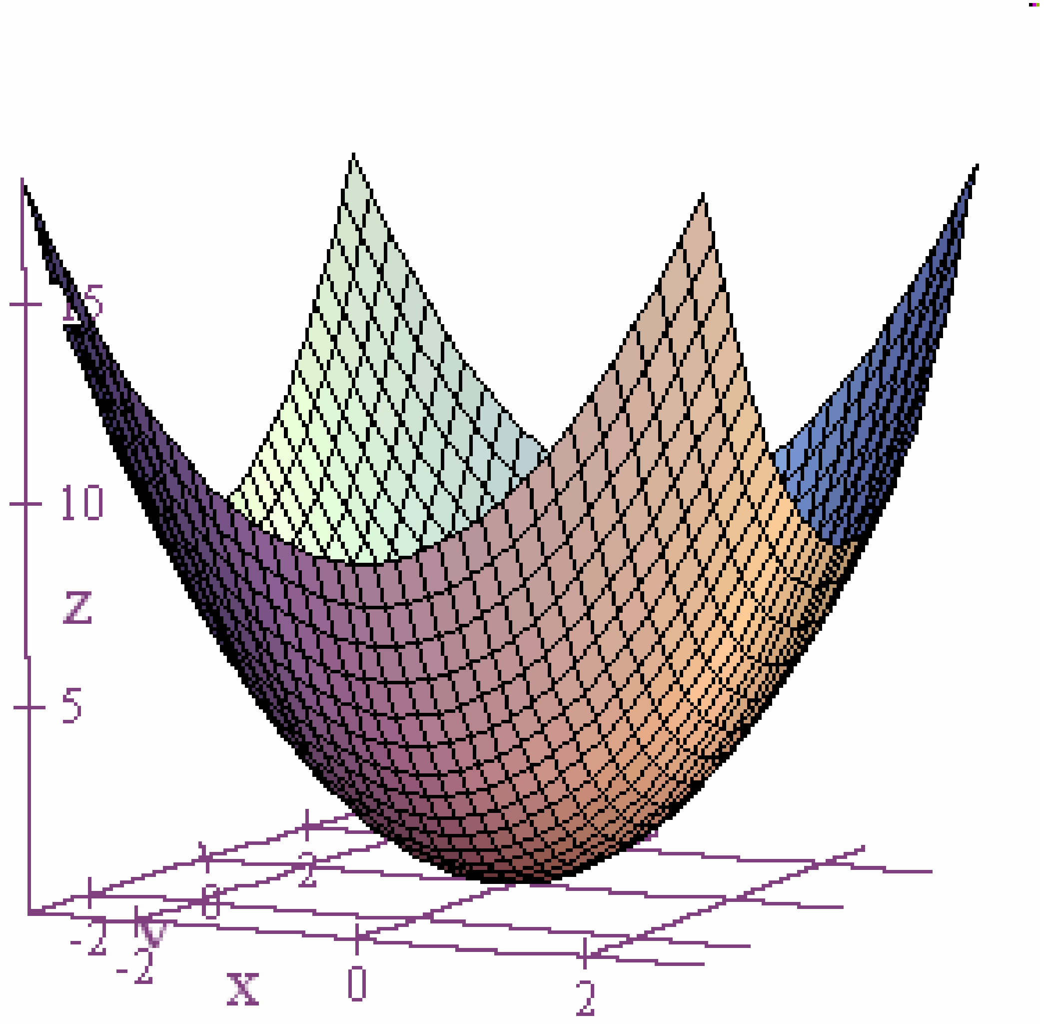
Algebraically:

$$z = f(x, y) = x^2 + y^2$$

# Numerically:

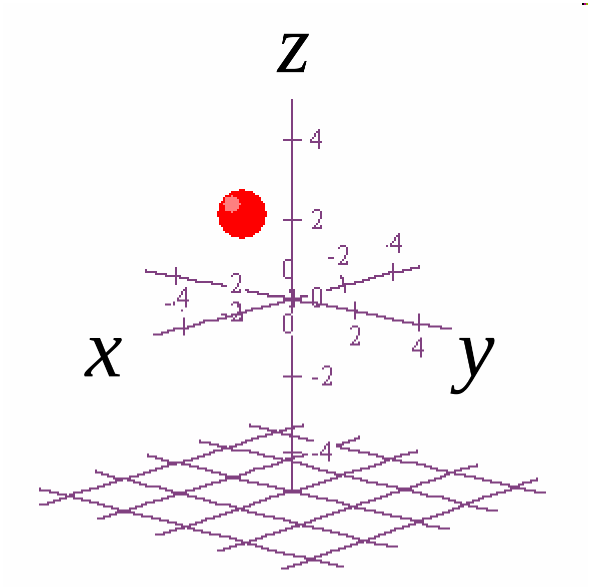
| x\y | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **-2** | 8 | 5 | 4 | 5 | 8 |
| **-1** | 5 | 2 | 1 | 2 | 5 |
| **0** | 4 | 1 | 0 | 1 | 4 |
| **1** | 5 | 2 | 1 | 2 | 5 |
| **2** | 8 | 5 | 4 | 5 | 8 |

# Or Graphically:



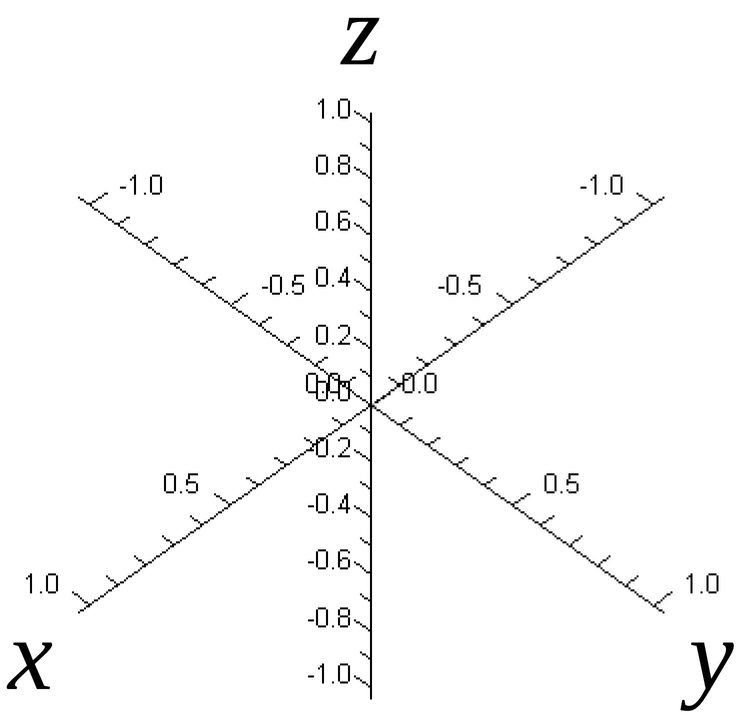
# PLOTTING POINTS

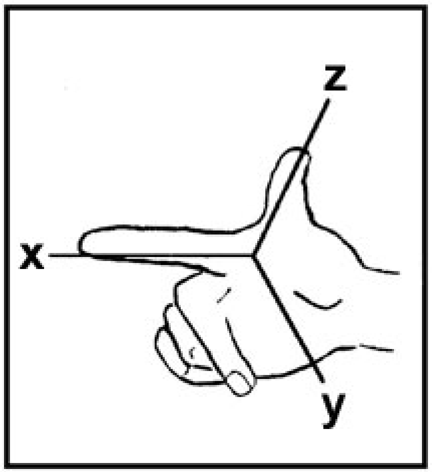
We can locate positions in 3-dimensional space by establishing an x-axis, y-axis, and z-axis, and then specifying an x-coordinate, y-coordinate, and z-coordinate for particular points.




$$(x, y, z) = (4, 2, 3)$$

This orientation called a right-hand coordinate system.



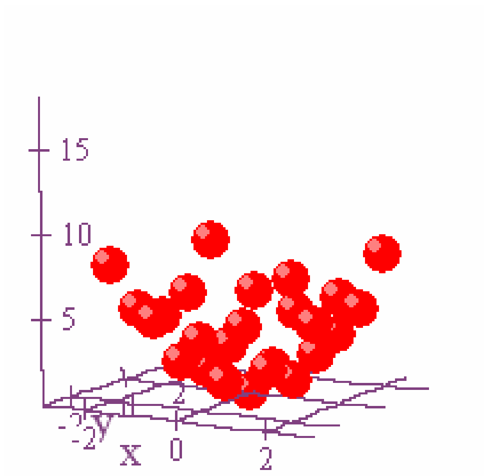



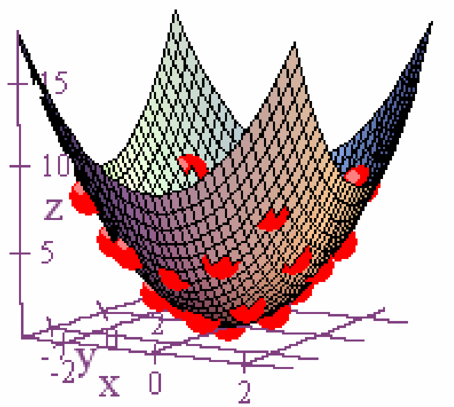
We can use the function below to generate the coordinates of points to plot.

$$z = f(x, y) = x^2 + y^2$$

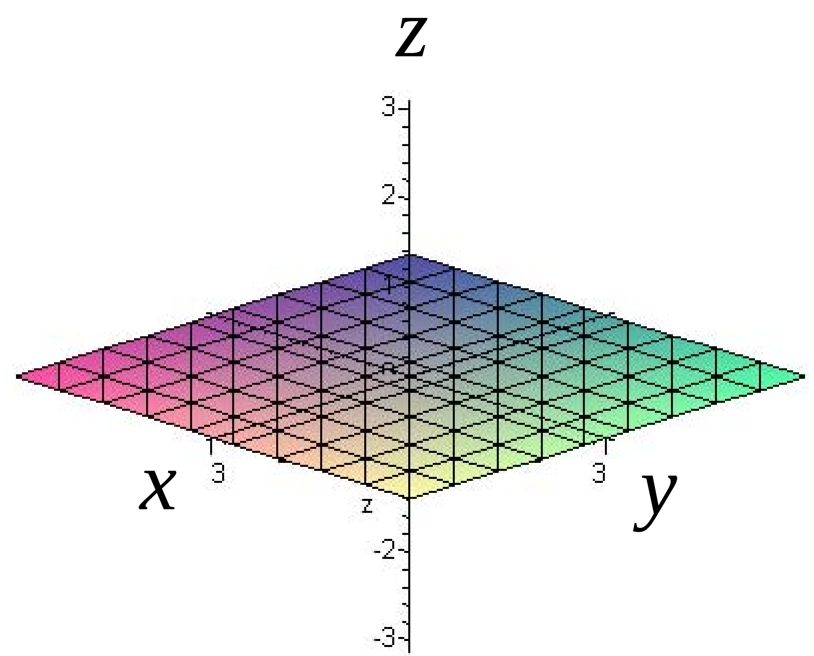
| **x\y** | **-2** | **-1** | **0** | **1** | **2** |
|---|---|---|---|---|---|
| **-2** | 8 | 5 | 4 | 5 | 8 |
| **-1** | 5 | 2 | 1 | 2 | 5 |
| **0** | 4 | 1 | 0 | 1 | 4 |
| **1** | 5 | 2 | 1 | 2 | 5 |
| **2** | 8 | 5 | 4 | 5 | 8 |

And from there it's just a matter of plotting points until the plot thickens!

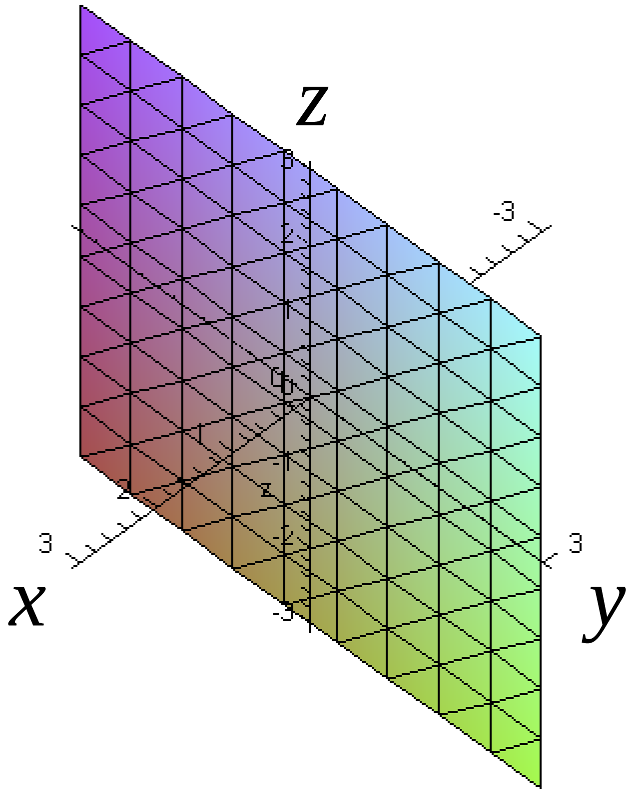


The graph of z=0 is the xy-plane.



The graph of x=0 is the yz-plane.



The graph of y=0 is the xz-plane.