The Directional Derivative

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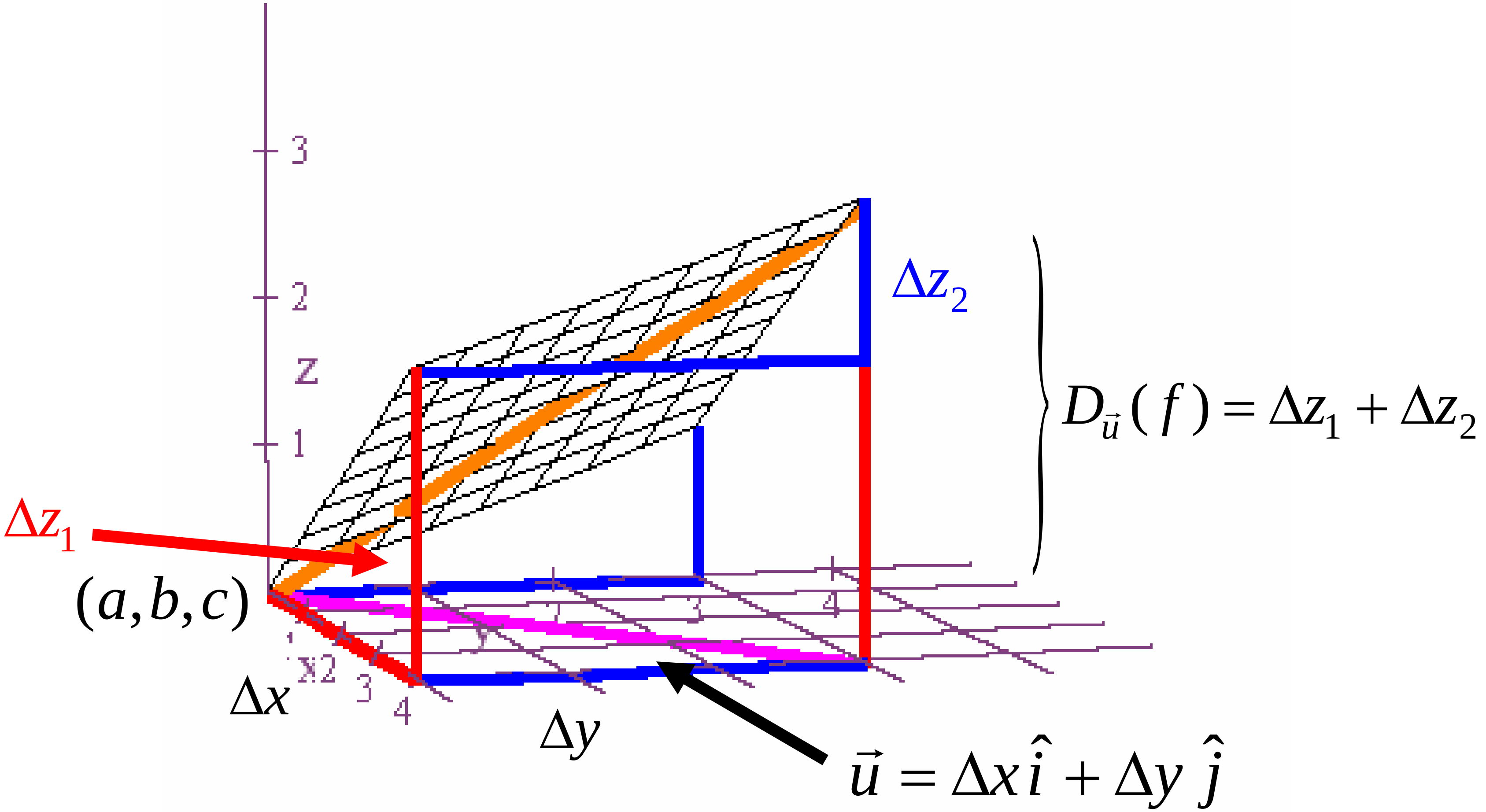
Suppose *z=f(x,y)* is a differentiable at the point *(a,b,c).*

Let *u* be a unit vector pointing in the direction in which we want to find the derivative.

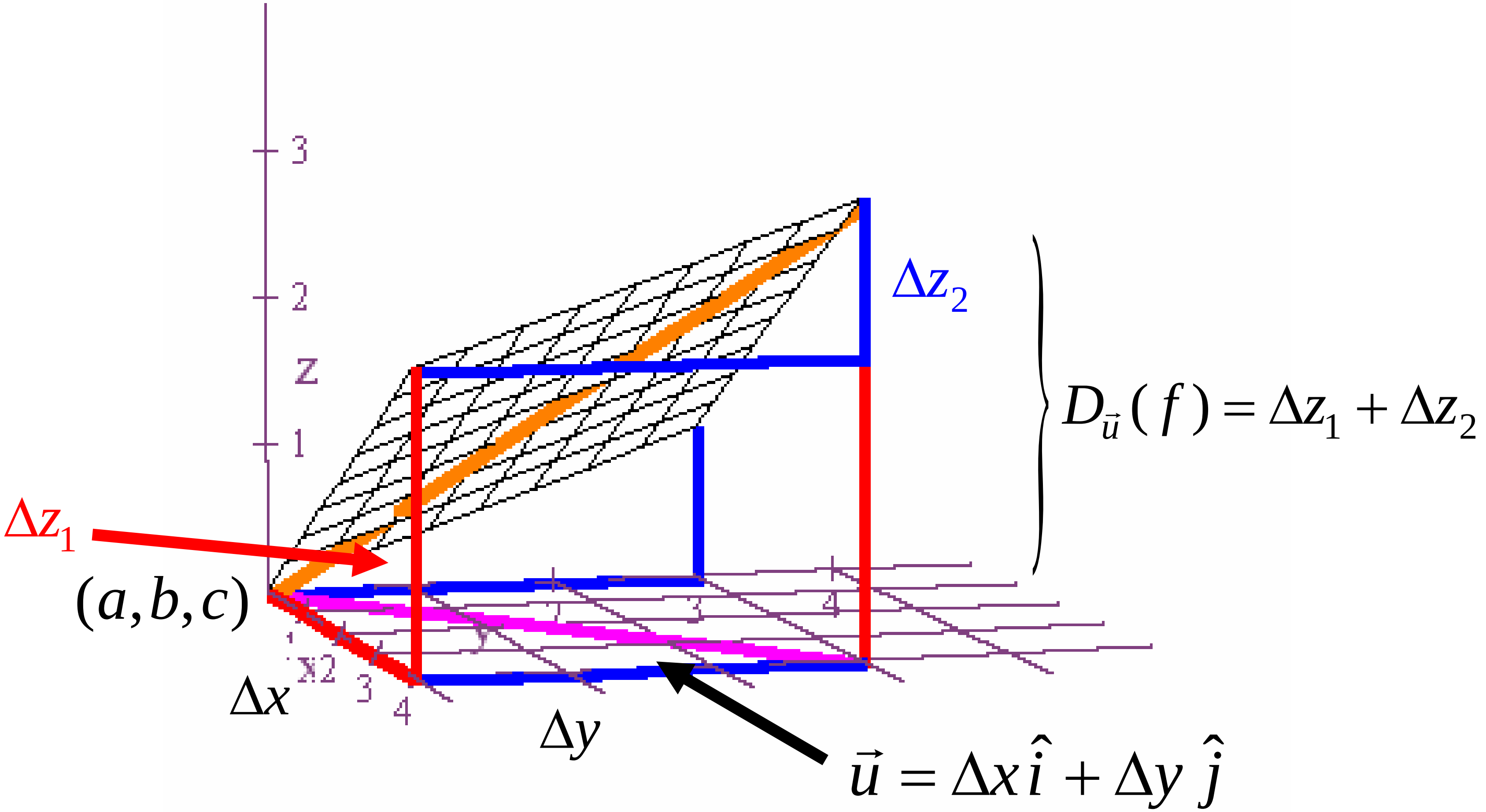
$$\vec{u} = \Delta x\,\hat{i} + \Delta y\,\hat{j}$$

$$directional\ derivative = D_{\vec{u}}(f)$$

The Directional Derivative



The Directional Derivative



$$D_{\vec{u}}(f) = \Delta z_1 + \Delta z_2 = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y$$

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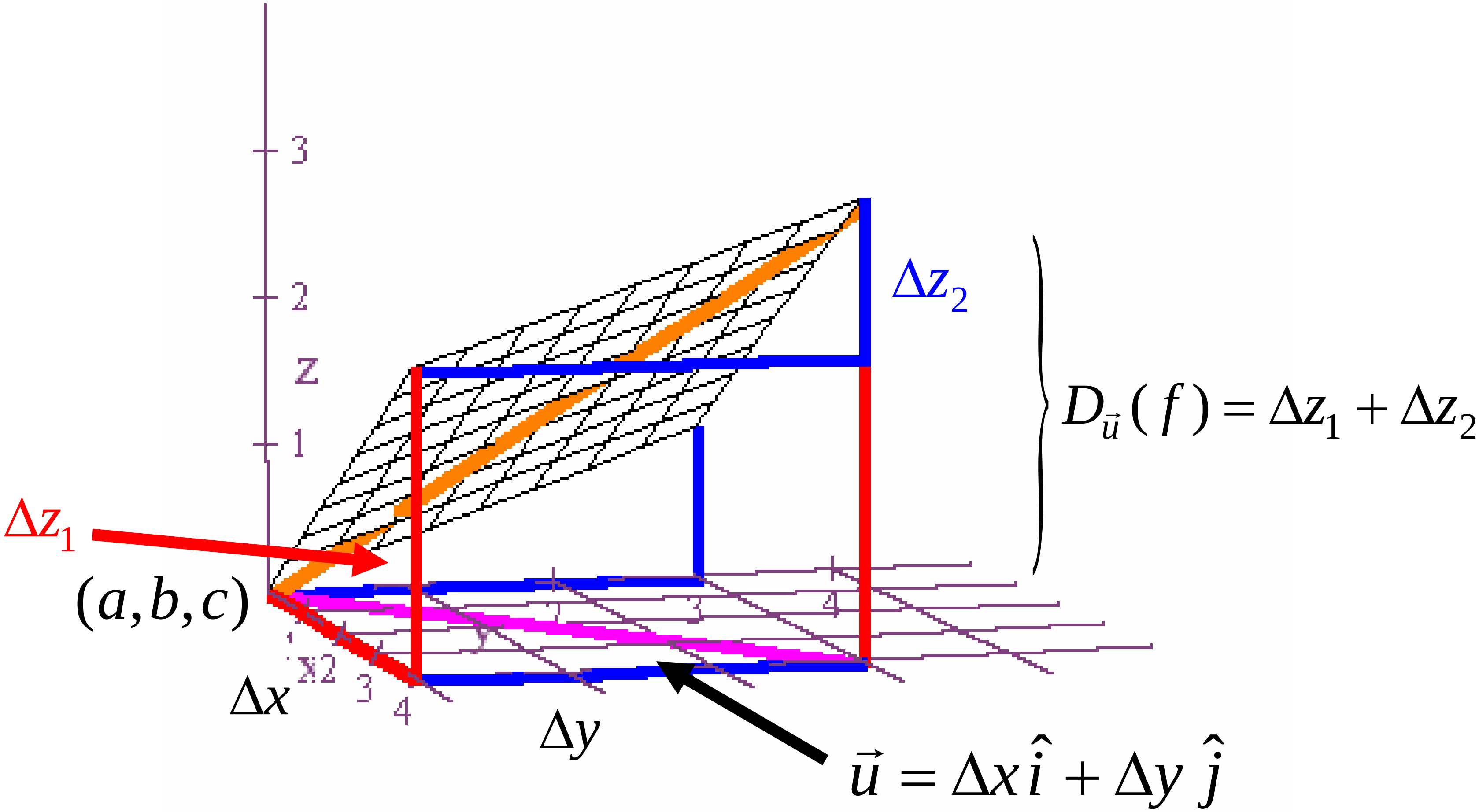

$$D_{\vec{u}}(f) = \Delta z_1 + \Delta z_2 = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y = \nabla f \cdot \vec{u}$$

A consequence is that $D_{\vec{u}}(f) = \nabla f \cdot \vec{u} = \left\| \nabla f \right\| \left\| \vec{u} \right\| \cos\theta = \left\| \nabla f \right\| \cos\theta$.

A consequence is that $D_{\vec{u}}(f) = \nabla f \cdot \vec{u} = \|\nabla f\| \|\vec{u}\| \cos\theta = \|\nabla f\| \cos\theta.$

In what direction defined by θ is $D_{\vec{u}}(f) = \|\nabla f\| \cos\theta$ at its maximum?

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$$\theta = 0° = 0 \; radians$$

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$$\theta = 180° = \pi \ radians$$

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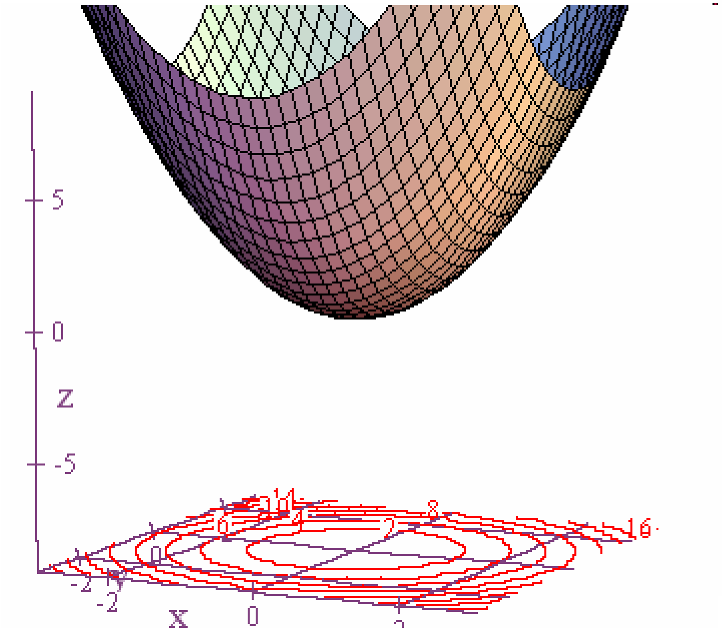
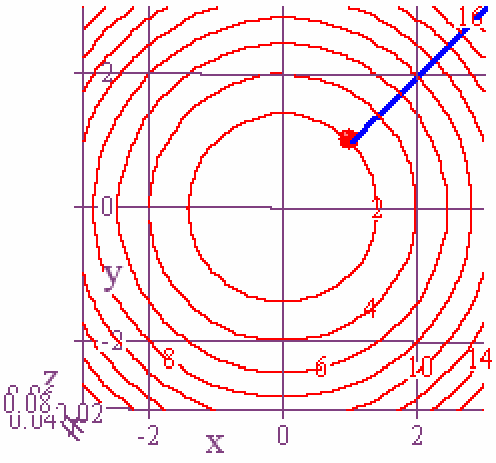
In what direction defined by θ is $D_{\vec{u}}(f) = \|\nabla f\| \cos\theta = 0$?

$$\theta = 90° = \frac{\pi}{2} \; radians$$

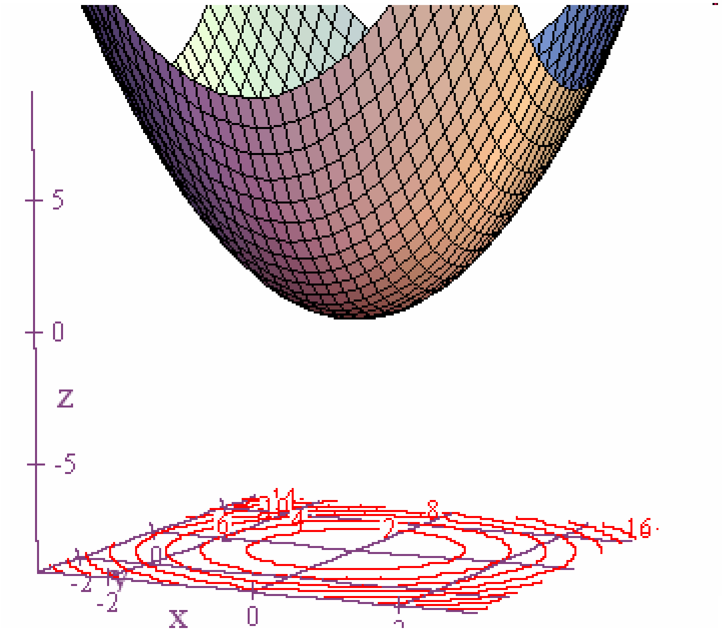
Now apply what we've done to the picture below.

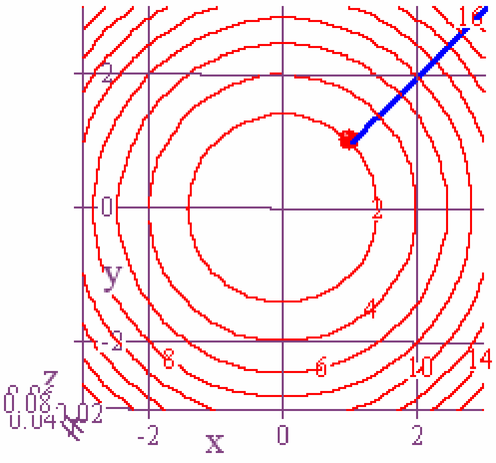




Now apply what we've done to the picture below.

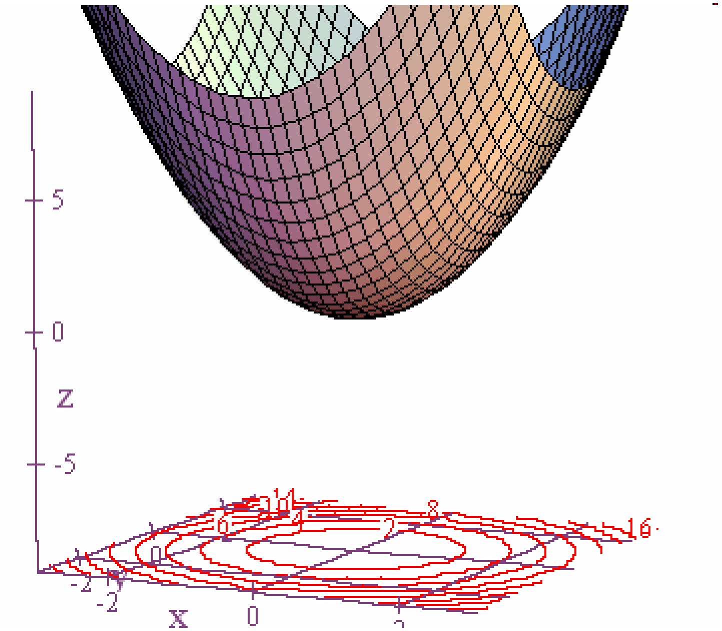
The gradient tells you what direction to move in in the *xy*-plane in order to make your output increase as quickly as possible. This direction maximizes the directional derivative.

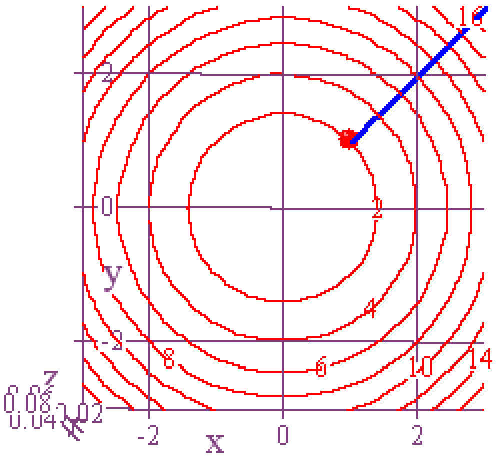




Now apply what we've done to the picture below.

The maximum rate of change at a point is equal to $\|\nabla f\|$.

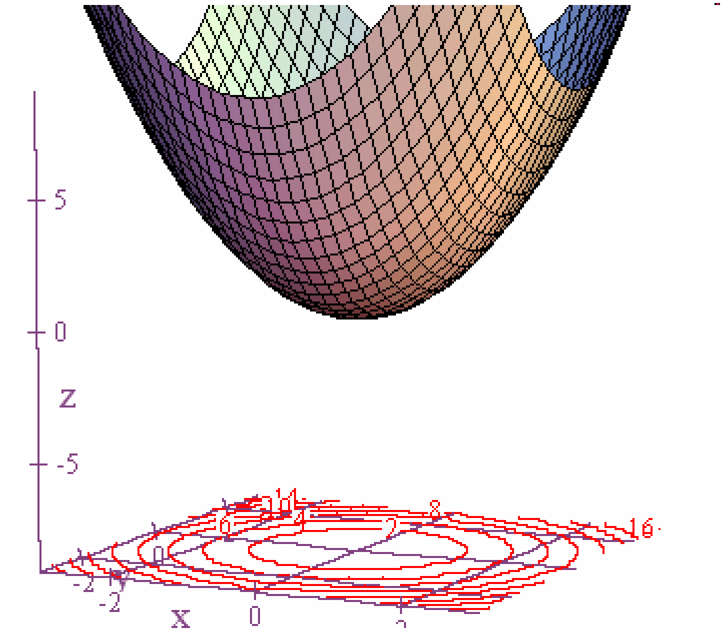


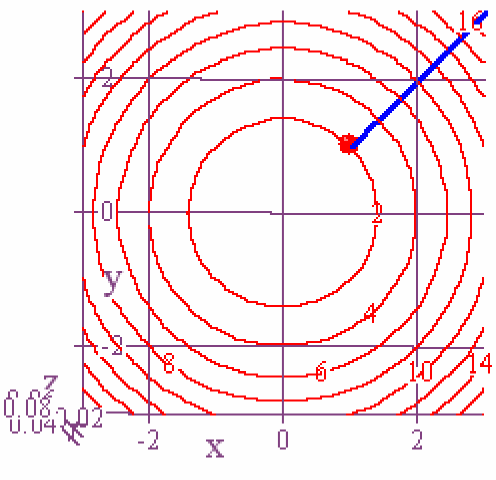


Now apply what we've done to the picture below.

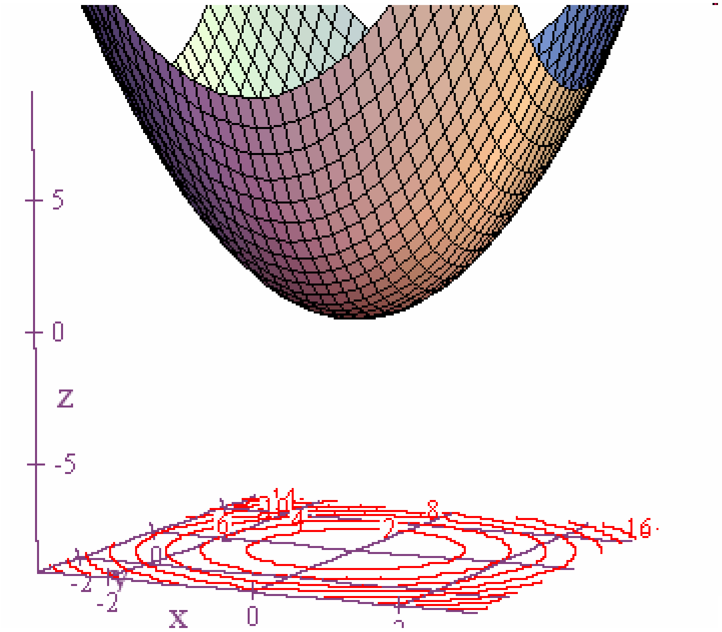
The maximum rate of change at a point is equal to $\|\nabla f\|$.

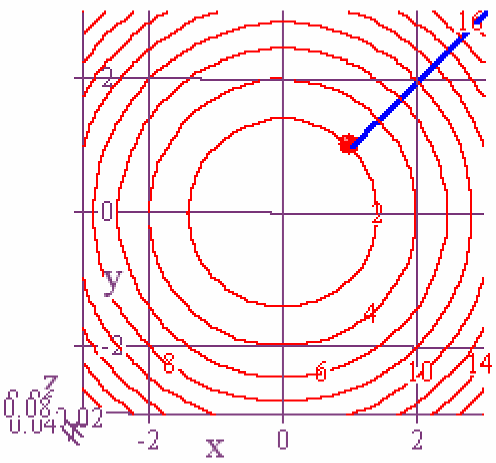
And the minimum rate of change is equal to -$\|\nabla f\|$.





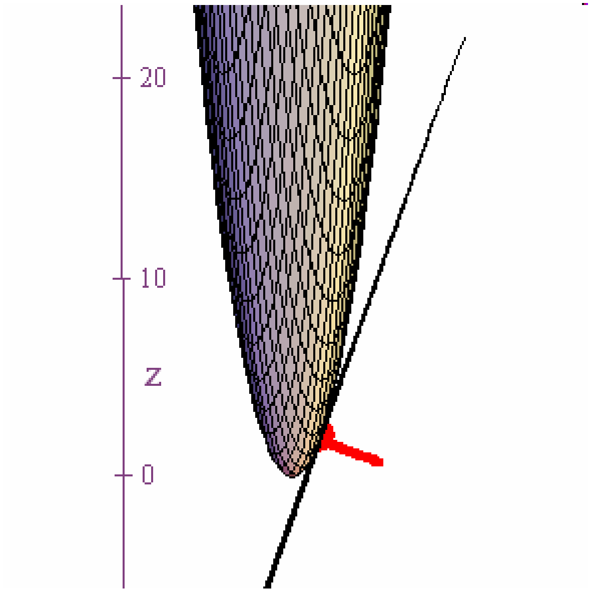
All of this applies, also, to functions of the form $w = f(x, y, z)$.





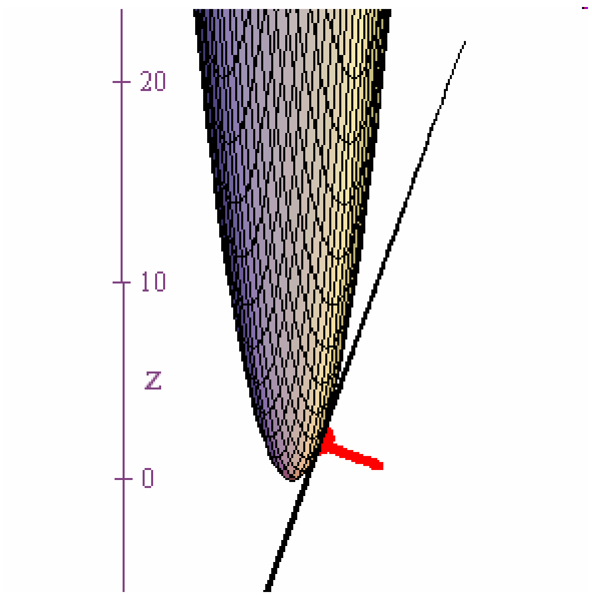
All of this applies, also, to functions of the form $w = f(x, y, z)$.

If you move in the direction of the gradient vector in 3-dimensional space, then your values for $w = f(x, y, z)$ will increase at their most rapid rate.



All of this applies, also, to functions of the form $w = f(x, y, z)$.

And the maximum rate of change is equal to $\|\nabla f\|$.



All of this applies, also, to functions of the form $w = f(x, y, z)$.

And the minimum rate of change is equal to $-\|\nabla f\|$.